

PUBLIC GOODS AND THE LAW OF $1/n$

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Abstract

Three features are often present in models of distributive politics: (i) individual legislators are assumed to care mainly or exclusively about the public projects that flow into their districts, (ii) the tax system that finances public projects is fixed and “decoupled” from the projects themselves — e.g., taxes are proportional to income or population; and (iii) the legislature is assumed adopt a norm of universalism, in which all projects proposed are passed (perhaps in one omnibus budget bill, or a small number of such bills). These assumptions lead to an oversupply of public projects, sometimes called “the law of $1/n$ ” (Weingast, Shepsle and Johnsen 1981). In this paper we demonstrate that the payoff function assumed for legislators in the literature is incorrect, at least for the case of pure public goods. Using a more general functional form we also establish the fragility of the “law of $1/n$ ” result. Our findings have implications for both theoretical and empirical studies of legislative organization and its effect on legislator incentives and spending outcomes.

1. Introduction

Beginning with Weingast (1979), a number of papers have analyzed various models of distributive politics in which: (i) individual legislators are assumed to care mainly or exclusively about the public projects that flow into their districts, (ii) the tax system that finances public projects is fixed and “decoupled” from the projects themselves — e.g., taxes are proportional to income or population; and (iii) the legislature is assumed adopt a norm of universalism, in which all projects proposed are passed (perhaps in one omnibus budget bill, or a small number of such bills). These assumptions lead to an oversupply of public projects, sometimes called “the law of $1/n$ ” (Weingast, Shepsle and Johnsen 1981). A number of papers attempt to test this “law” and find support for it (e.g., Gilligan and Matsusaka 1995, 2001; Bradbury and Crain 2001; Baqir 2002; Bradbury and Stephenson 2003*a,b*). Pettersson-Lidbom (2004) finds that a “reverse law of $1/n$ ” holds and calls for a theoretical explanation for this finding.¹ Our paper suggests one possibility.

In this paper we demonstrate that the payoff function assumed for legislators in the literature is incorrect, at least for the case of pure public goods. Using a more general functional form we also establish the fragility of the “law of $1/n$ ” result. Our findings have implications for both theoretical and empirical studies of legislative organization and its effect on legislator incentives and spending outcomes.

2. The Standard Formulation

Previous work typically specifies legislators’ payoffs as follows.² Let the total population of a nation be divided into n equally-sized districts. Let X be a publicly provided project of “size” X . Let $C(X)$ be the total costs of the project. Let $B(X)$ be the total benefit received by the citizens in the district where the project is located (assume no spillovers). Assume

¹The term “reverse law of $1/n$ ” is ours.

²Weingast, Shepsle and Johnsen (1981) consider a slightly more general version of this model with tax shares not necessarily equal. The case we consider here is the most typical implementation of their model.

$C' > 0$, $C'' > 0$, $B' > 0$ and $B'' \leq 0$. Assume full cost-sharing of all projects, and assume taxes are equal for all citizens.

Since all citizens in each district are identical, each legislator's payoff should be equal to the payoff of her representative citizen. Alternatively, each legislator's payoff could be set equal to the total payoff of all citizens in her district. Let us take this second approach. Suppose district i receives a project of size X_i , and all other districts receive projects of size X . Then the payoff for the legislator representing district i is defined as

$$\Pi_i(X_i, n) = B(X_i) - C(X_i)/n - C(X)(n-1)/n . \quad (1)$$

This formulation leads instantly to the "law of 1/n." Assume each legislator chooses the size of her district's project, X_i , taking all other districts' projects as fixed. Differentiating with respect to X_i yields the first-order condition $B'(X_i^*) = C'(X_i^*)/n$. Differentiating this first-order condition totally with respect to n yields

$$\frac{\partial X_i^*}{\partial n} = \frac{B'(X_i^*)}{C''(X_i^*) - nB''(X_i^*)} > 0 . \quad (2)$$

Thus, each district's project size is increasing in the number of districts. The number of projects is also increasing in the number of districts, since each district receives a project. So, total government spending, $nC(X_i^*)$, is clearly increasing in the number of districts.

Perotti and Kontopoulos (2002, 195) state the typical intuition for this result:

"Individual groups and hence, indirectly, their representatives in the fiscal policy decision-making process benefit from specific types of expenditures; by contrast, because of basic constitutional principles, typically taxation falls on large segments of the population and cannot be easily targeted. Hence, each group and their representatives fully internalize the benefits of the expenditures they propose; however, they internalize only a fraction 1/n of the costs the associated revenues impose on the whole economy. Clearly this fraction falls as the number n of groups and their representatives increases. Thus, a larger number of participants in the process leads to a larger total expenditure. ... This intuition is straightforward enough."

So what is the problem? Let us back up and begin again from first principles.

Let N be the total number of citizens in a country, let n be the number of districts (all equal in population), and let $m = N/n$ be the number of citizens in each district. As before, let X denote a publicly provided project of “size” X . Let $C(X)$ be the total costs of the project, in dollars. Let $b(X, m)$ be the benefit received by each citizen from the project, and let $B(X, m)$ be the aggregate benefits in the district.³

If projects are pure private goods, then we can think of the “size” of a project simply as the number of “units” of project, these units being divided equally among the m citizens. In this case, $b(X, m) = \tilde{b}(X/m)$ and $B(X, m) = \tilde{b}(X/m)m$, with $\tilde{b}' > 0$ and $\tilde{b}'' \leq 0$.

On the other hand, if projects are pure public goods, then each citizen receives the “full” benefits of a project regardless of m . In this case, $b(X, m) = \tilde{b}(X)$ and $B(X, m) = \tilde{b}(X)m$, with $\tilde{b}' > 0$ and $\tilde{b}'' \leq 0$.

Suppose project size in district i is X_i , and suppose project size in all other districts is X . Assume full cost-sharing, and assume taxes are equal for all citizens. Then the payoff to a representative citizen in the district under consideration is

$$\hat{\pi}_i(X_i, m, n) = b(X_i, m) - C(X_i)/nm - C(X)(n-1)/nm \quad (3)$$

$$= b(X_i, m) - C(X_i)/N - C(X)(n-1)/N . \quad (4)$$

The denominator of the cost terms is $N = nm$, because the citizens in district i collectively pay $(1/n)$ th of the cost of all projects, and this cost is split evenly across the m citizens in the district; so, each citizen pays $(1/N)$ th of the total cost of the projects in all districts. Alternatively, the aggregate payoff to the citizens in district i is equal to

$$\hat{\Pi}_i(X_i, m, n) = B(X_i, m) - C(X_i)/n - C(X)(n-1)/n . \quad (5)$$

³Knight (2003, 8) solves a related model, but he also adopts the standard intuition: “As the number of districts increases, the common pool problem becomes more severe, leading to an increase in aggregate spending.”

In the pure public goods case, the payoff to the representative citizen in district i and the aggregate payoff to all citizens in district i are, respectively,

$$\hat{\pi}_i(X_i, m, n) = \tilde{b}(X_i) - C(X_i)/N - C(X)(n-1)/N, \quad \text{and} \quad (6)$$

$$\hat{\Pi}_i(X_i, m, n) = \tilde{b}(X_i)m - C(X_i)/n - C(X)(n-1)/n. \quad (7)$$

Either of these is a valid payoff function for the legislator representing district i .

What is the difference between equation (7) and the standard formulation, equation (1)? The cost terms are the same. But the benefit terms are not. *The standard formulation misses the fact that if total population N is fixed, then it is impossible to change the number of districts, n , without also changing the population in each district, m , since $N = nm$. And, in the case of public goods, changing m automatically changes the aggregate net benefits to the citizens of district i — and, therefore, the payoff of legislator i .*

Alternatively, consider the problem from the point of view of the representative citizen in district i . Dividing equation (1) by m gives the payoff of the representative citizen in the standard formulation,

$$\pi_i(X_i, m, n) = B(X_i)/m - C(X_i)/N - C(X)(n-1)/N. \quad (8)$$

Comparing this with equation (6) we see that the cost terms are again the same. But the benefit terms are not. Rather, equation (8) is much closer to the private goods case than the public goods case. Indeed, if B is linear with slope β then it corresponds exactly to a private goods case, with $b(X, m) = \beta X/m$.

Weingast, Shepsle and Johnsen's (1981) "law of 1/n" states that the inefficiency of projects is growing in the number of districts (implicitly holding population constant). Since universalism is at work, this implies that spending must be increasing in the number of districts. Does the "law of 1/n" still hold in the pure public goods case? It is clear from equation (6) that the answer is no. Differentiating equation (6) with respect to X_i yields the

first-order condition $\tilde{b}'(X_i^*) = C'(X_i^*)/N$. Since n does not appear explicitly in this equation

$$\frac{\partial X_i^*}{\partial n} = 0 . \quad (9)$$

Thus, the number of districts has no effect on project size, and the “law of 1/n” as stated is incorrect for pure public goods.

Why is project size independent of the number of districts? Examining aggregate district payoffs makes this clearer. Differentiate equation (7) to obtain the first-order condition $\tilde{b}'(X_i^*)m = C'(X_i^*)/n$. Substituting for m , this becomes $\tilde{b}'(X_i^*)N/n = C'(X_i^*)/n$. So, increasing n produces two competing forces, which just happen to cancel one another. On one hand, increasing n exacerbates the “tragedy of the commons” problem inherent under the norm of universalism plus full cost sharing. This is captured by the term on the right-hand side of the equality — it is a force that increases X_i . On the other hand, as n increases m decreases, and since the goods are pure public goods the benefits to increasing X_i fall — there are fewer people in each district to share in the public good. This is captured by the term on the left-hand side of the equality — it is a force that decreases X_i . The more typical interpretation of the “law of 1/n” — that spending is increasing in the number of districts — holds in this case because the number of projects is increasing in n , not because project sizes are increasing in n . And in fact, even this result is not robust, as the next section demonstrates.

3. Crowding, Deadweight Costs, and Partial Cost-Sharing

Consider now a formulation that allows us to incorporate crowding, deadweight costs of taxation, and partial sharing of project costs. To simplify the analysis we adopt specific functional forms, but these are not critical for the results. Let total benefits from a project of size X be $B(X, m) = X^\alpha m^\beta$, where $0 < \alpha < 1$. Per capita benefits then can be written as $b(X, m) = X^\alpha m^{\beta-1}$. Crowding occurs when the addition of more individuals reduces the

benefits each individual receives from a project. Formally, this occurs when $\beta < 1$. The case of pure public goods can be represented by setting $\beta = 1$, and the case of pure private goods can be represented by setting $\beta = 1 - \alpha$.

Suppose project costs are linear in project size, so $C(X) = X$. Let s be the fraction of each project's costs that are shared equally by all districts, and let $1 - s$ be the fraction that is paid by the district that enjoys the project's benefits. Let X_i be project size in district i , and assume that project size in all other districts is X . Then each citizen in district i pays taxes of $t_i = (1 - s)X_i/m + s(X_i + (n - 1)X)/N = [(n - ns + s)X_i + (ns - s)X]/N$.

To capture the deadweight costs of taxation, assume that if the amount of taxes paid by a citizen is t , then the total cost borne by the citizen is t^θ , where $\theta \geq 1$. If $\theta > 1$, then taxes carry deadweight costs.

We can rewrite equation (4) as:

$$\hat{\pi}_i(X_i, m, n) = X_i^\alpha m^{\beta-1} - [(n - ns + s)X_i + (ns - s)X]^\theta (1/N)^\theta . \quad (10)$$

Differentiating with respect to X_i and rearranging yields the first-order condition

$$\left(\frac{\alpha}{\theta}\right) \left(\frac{N^{\beta+\theta-1} n^{1-\beta}}{n - ns + s}\right) = [(n - ns + s)X_i + (ns - s)X]^{\theta-1} X_i^{1-\alpha} . \quad (11)$$

Since all districts are identical, in equilibrium they all choose projects of the same size, so $X_i = X = X_i^*$. Substituting and rearranging, in (11) becomes

$$X_i^* = (\alpha/\theta)^{\frac{1}{\theta-\alpha}} (N)^{\frac{\beta+\theta-1}{\theta-\alpha}} \left[\frac{n^{2-\beta-\theta}}{n - ns + s}\right]^{\frac{1}{\theta-\alpha}} . \quad (12)$$

Differentiating (12) with respect to n yields the condition

$$\frac{\partial X_i^*}{\partial n} > 0 \quad \text{if and only if} \quad \beta + \theta < \frac{n - ns + 2s}{n - ns + s} . \quad (13)$$

In the case of full cost-sharing, $s = 1$ and (13) simplifies to the following:

Proposition 1 *Under full cost-sharing, project sizes are increasing [decreasing, constant] in n when $(\beta + \theta) < [>, =]2$.*

For the simple case of a pure public good and no deadweight costs of taxation, $\beta = 1$ and $\theta = 1$. Then $\beta + \theta = 2$, demonstrating that the “law of $1/n$ ” fails to hold for this case. Some crowding is a necessary condition for the “law of $1/n$ ” to hold; the result obtains whenever there is sufficient crowding (i.e., β is not too close to 1) and deadweight costs are not too high (i.e., θ is close to 1), because in those cases increasing the number of districts will mitigate the crowding problem. On the other hand, with a sufficiently public good (i.e., β near 1) and sufficiently large deadweight costs of taxation (i.e., $\theta > 1$), we will have $\beta + \theta > 2$ and therefore a law that is the *opposite* of the “law of $1/n$.” Deadweight costs cause this because for any fixed project size, the marginal cost of taxation increases with n , which tends to reduce the optimal value of X_i .

With partial cost-sharing “the law of $1/n$ ” is even more likely to fail. Interestingly, if we begin with a fairly large number of districts, then even with “nearly full” cost-sharing we often find that project size *decreases* as the number of districts increases further. The number of districts, n , looms large in the calculations. The table below gives several examples.

Maximum value of $\beta + \theta$ such that $\partial X_i^*/\partial n > 0$		
n	s	value
20	.95	1.49
20	.90	1.31
50	.95	1.28
50	.90	1.15
100	.95	1.16
100	.90	1.08

In all of these cases, $\partial X_i^*/\partial n < 0$ even for values of β well below 1. Moreover, for any $s < 1$, as n gets large, the maximum value in column 3 of the table goes to 1. In other words, as the number of districts increases, it is harder for the law of $1/n$ result to hold.

We can perform the same analysis for total spending. Total spending across all districts

is

$$nX_i^* = (\alpha/\theta)^{\frac{1}{\theta-\alpha}} (N)^{\frac{\beta+\theta-1}{\theta-\alpha}} \left[\frac{n^{2-\beta-\alpha}}{n-ns+s} \right]^{\frac{1}{\theta-\alpha}}. \quad (14)$$

Differentiating (14) with respect to n yields the condition

$$\frac{\partial X_i^*}{\partial n} > 0 \quad \text{if and only if} \quad \alpha + \beta < \frac{n-ns+2s}{n-ns+s}. \quad (15)$$

In the case of full cost-sharing, $s = 1$ and (15) simplifies to $\alpha + \beta < 2$. This always holds, since $\alpha < 1$ and $\beta \leq 1$.

Proposition 2 *Under full cost-sharing, total spending is always increasing in n .*

This result demonstrates the problem with using total spending to analyze whether legislator decision-making is affected by the number of districts. Regardless of whether legislators want to increase, decrease, or leave unchanged their project request, total spending will be increasing in the number of districts.

Moreover, under partial cost-sharing, even this version of the “law of 1/n” may fail. This is clear from the table above, since the values in the last column give the maximum values for $\alpha + \beta$ such that $\partial(nX_i^*)/\partial n > 0$. Thus, if we begin with a fairly large number of districts, then total spending may be *decreasing* in the number of districts even with “nearly full” cost-sharing. For example, in all of the cases shown in the table, all we need is $\alpha > .75$ and $\beta > .75$.

4. Conclusion

Several important results emerge from the simple setup above:

1. The “law of 1/n,” as originally presented by Weingast, Shepsle, and Johnsen (1981), states that the inefficiency of projects is increasing in the number of districts. We have

shown that this result does not hold for pure public goods. A weaker version, that spending is increasing in the number of districts, holds under the assumption of full cost-sharing, but not for the reasons typically argued in the literature.

2. By incorporating crowding and deadweight costs of taxation, we have demonstrated that the “law of $1/n$ ” result requires some crowding and sufficiently small deadweight costs of taxation. A “reverse law of $1/n$ ” can hold for sufficiently public goods and large deadweight costs of taxation. In these cases, increasing the number of districts reduces the number of individuals in each district (holding N fixed), which reduces the benefit of the government-provided good and also increases the cost of generating revenue to fund it.
3. Partial cost-sharing causes the deadweight costs of taxation to loom larger, making the “law of $1/n$ ” less likely to hold even when costs are nearly fully taken out of a common pool.
4. Total spending is a misleading measure of the effects of districting on legislator decision making. The reason is that the number of districts influences both (a) the projects legislators select and (b) the number of projects. Both have an impact on spending, with sometimes countervailing impacts.

This paper has important implications for formal and empirical research. On the formal side, it suggests that the effects of districting depend crucially on the types of goods being provided by government. For a pure public good, the “law of $1/n$ ” fails to hold, and the result holds under only limited circumstances for other types of goods. On the empirical side, it suggests that more careful attention should be paid to how one uses models of legislative organization to motivate data analysis. A natural extension of this research is to examine the impact of district heterogeneity on legislator preferences for publicly provided goods.⁴

⁴See Crain (1999) and Knight (2003) for a discussion of this issue.

References

- Baqir, Reza. 2002. "Districting and Government Overspending." *Journal of Political Economy* 110: 1318–1359.
- Bradbury, John Charles, and E. Frank Stephenson. 2003a. "The Law of $1/n$: A New Test." Working Paper: Department of Economics, University of the South and Berry College.
- Bradbury, John Charles, and E. Frank Stephenson. 2003b. "Local Government Structure and Public Expenditures." *Public Choice* 115: 185–198.
- Bradbury, John Charles, and W. Mark Crain. 2001. "Legislative Organization and Government Spending." *Journal of Public Economics* 82: 309–325.
- Crain, W. Mark. 1999. "Districts, Diversity, and Fiscal Biases: Evidence from the American States." *Journal of Law and Economics* 42: 675–698.
- Gilligan, Thomas W., and John G. Matsusaka. 1995. "Deviations From Constituent Interests: The Role of Legislative Structure and Political Parties in the States." *Economic Inquiry* 33: 383–401.
- Gilligan, Thomas W., and John G. Matsusaka. 2001. "Fiscal Policy, Legislature Size, and Political Parties: Evidence from the First Half of the Twentieth Century." *National Tax Journal* 54: 57–82.
- Knight, Brian. 2003. "Common Tax Pool Problems in Federal Systems." Working paper: Department of Economics, Brown University.
- Perotti, Roberto, and Yianos Kontopoulos. 2002. "Fragmented Fiscal Policy." *Journal of Public Economics* 86: 191–222.
- Petterson-Lidbom, Per. 2004. "Does the Size of the Legislature Affect the Size of Government? Evidence from Two Natural Experiments." Working paper: Department of Economics, Stockholm University.

Weingast, Barry R. 1979. "A Rational Choice Perspective on Congressional Norms." *American Journal of Political Science* 23: 245–262.

Weingast, Barry R., Kenneth A. Shepsle, and Christopher Johnsen. 1981. "The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics." *Journal of Political Economy* 89: 642–664.