Labor Economics, Problem Set 2

This problem set is due on or before Wednesday November 21, 4 p.m..

Please answer the following questions:

Exercise 1 *A worker has utility $U = log(s) - e$ where s is reward and e is effort which can be equal to either 0 or 1. The effort choice of the worker is not observed by the risk-neutral principal who only cares about net profits. The worker on the other hand needs to be given an ex ante expected utility level equal to u^*. The probability density for revenue, x,* defined only for $x \geq 1$, *is $f(x) = 2x^{-3}$ if $e = 0$ and $f(x) = x^{-2}$ if $e = 1$.*

1. *Suppose low effort is preferred, determine the optimal contract.*
2. *Suppose high effort is preferred, determine the optimal contract. (Hint: the optimal contract will have the form $s(x) = A - B/x$).*
3. *Write down the condition for high effort to be preferred to low effort.*

Exercise 2 *A worker can choose $e = 0$ or $e = 1$. The worker's utility is*

$$u(w) - Ke,$$

where w is the wage, $K > 0$ and $u(w)$ is concave. The worker cannot be paid less than zero, so $w \geq 0$ and has a reservation utility equal to 0. If $e = 0$, the project has a probability of success equal to p, in which case it produces a revenue of y. If it's unsuccessful, it produces a revenue of zero. If $e = 1$ the project has a probability of success equal to $q > p$.

1. *Write the optimization problem of the principal assuming that she wants to implement $e = 1$.*
2. *Determine the wage contract and indicate the conditions under which the worker needs to be paid an "efficiency wage".*
3. *Determine whether the principal prefers $e = 1$ to $e = 0$.*

Exercise 3 *Consider the following career concerns model. The world lasts two periods. All firms and workers are risk neutral and there is no discounting. Workers are high or low ability, with ability denoted by $\eta \in \{\eta^H, \eta^L\}$, with $\eta^H > \eta^L$. The fraction of high ability workers in the population is $p \in (0, 1)$. Worker ability is observed neither by the worker nor by the firms in the market. Each worker chooses an effort $a \in \mathbb{R}_+$ at each date, and with probability $q(\eta, a) \in (0, 1)$, he generates high output $Y^h > 0$ and with the complementary probability, he generates low output Y^l, which we normalize to $Y^l = 0$ Assume that $q(\eta, a)$ is continuous, increasing and differentiable in a and increasing in η. The output level of each worker is publicly observed, but his effort level is not observed by potential employers. After the first period output level, Y_1 is realized, a large number of firms compete a la Bertrand to hire the workers. Finally, assume that workers have a continuous, differentiable, and convex cost of effort, $c(a)$.*

1. *Define a Perfect Bayesian Equilibrium for this game.*
2. *Show that in period 2, a worker will be paid*

   $$w_2(Y_1) = \pi(Y_1)\, q\left(\eta^H, 0\right) Y^h + (1 - \pi(Y_1))\, q\left(\eta^L, 0\right) Y^h,$$

   where $\pi(Y_1)$ is the probability that the market assigns to the worker being high ability after observing his output level $Y_1 \in \{Y^h, Y^l = 0\}$ in the first period.
3. *Suppose that all workers choose effort $\bar{a}$ in the first period and derive $\pi(Y_1)$ from Bayes's rule.*

4. *Given $w_2(Y_1)$ and $\pi(Y_1)$, derive the best response first period effort a of workers. Show that in equilibrium this effort must satisfy $a = \bar{a}$.*

5. *Provide conditions such that a symmetric pure strategy equilibrium exists. Can there be multiple equilibria? Provide an economic intuition.*

6. *Suppose that a unique symmetric pure strategy equilibrium exists. What is the impact of an increase in Y^h on equilibrium effort level? How does this effort depend on the form of the function $q(\eta, a)$? Can you relate this to any real-world labor market facts?*

7. *Define the "first-best" effort level. Can the equilibrium level of effort be greater than the first-best effort?*

8. *What is the difference of this model from the Holmstrom's baseline career concerns model? What are the advantages and disadvantages of this model?*

Exercise 4 *Consider the Shapiro-Stiglitz model where workers and firms are infinitely lived and risk-neutral, both with discount rate r. Effort costs e, and without effort there is no output produced. There are N firms each with production function $A.F(L)$ which is increasing and strictly concave where L is the number of workers employed by the firm who exert effort. There is an exogenous separation rate equal to b, and unemployed workers get disutility of leisure (and benefits) equal to z. Unemployed workers are randomly allocated to new job openings (which are due to separations). Firms decide what wage to offer to their workers.*

Workers who shirk (do not exert effort) are caught with probability q. The difference from the standard model is that q is chosen by the firm. It costs $C(q)$ per worker (thus a total of $C(q)L$) to choose a level of monitoring equal to q.

1. *First write the Bellman equations for given q and derive the incentive compatibility condition (or the no shirking constraint).*

2. *Now find a first-order condition to determine the optimal level of q for a firm (Hint: be careful here, a common mistake is not to distinguish between the "q" of the firm in question, say q_i, and the "q" of all other firms which enters through V^U and which is obviously not controlled by the firm).*

3. *Show that an increase in A, which reduces unemployment leaves q unchanged. Explain this result. Is it counter-intuitive?*

4. *How would you modify the model so that changes in A have an impact on q. Outline, if you can, possible ways of and generating the prediction that $dq/dA > 0$ and that $dq/dA < 0$.*

5. *Informally discuss whether $C(q)L$ as the cost of monitoring is plausible. In particular, would $C(q)$ be better? What would change in the model if instead of "q" we had workers supervising other workers (Hint: think of wages as costs)?*

Exercise 5 *Consider a version of the Shapiro-Stiglitz model where workers and firms are infinitely lived (in continuous time) and risk-neutral, both with discount rate r. Workers choose between high and low effort, and high effort costs e. Total population of workers is normalized to 1. There are N firms each with production function $AF(H_i)$, where F is increasing and strictly concave and H_i denotes the efficiency units of labor by the employees of the firm in question. Assume that*

$$H_i = [\lambda_i + (1 - \lambda_i)\alpha] L_i$$

where L_i is the total employment of this firm, λ_i is the fraction of employees exerting high effort (with the remainder exerting low effort) and $\alpha \in (0, 1)$. Also assume that $F'(H) > 0$ for all H.

There is an exogenous separation rate equal to b, and unemployed workers receive unemployment benefits equal to z (and no additional utility from leisure). Unemployed workers are randomly allocated to new job openings (which are due to separations). Firms decide what wage to offer to their workers. Workers who exert low effort are detected at the flow rate q, and the firm decides whether to keep them or fire them.

1. *First write the Bellman equations and derive the incentive compatibility condition (the no shirking constraint, which makes workers prefer to exert high effort) in terms of the value unemployment, V_U (here and throughout, you can impose steady state to simplify things). Explain why a firm would always fire a worker caught shirking, and why it would never pay a higher wage than the one that makes the incentive compatibility condition hold as equality (Hint: what about the participation constraint?).*

2. *Characterize the steady state equilibrium levels of output and employment assuming that firms always want to induce high effort from their employees (Hint: solve for V_U, and obtain an implicit equation in the level employment L as a function of parameters). What is the effect of the unemployment benefit, z, on employment and wages?*

3. *Take V_U as given, and determine the level of wage that the firm would pay to its employees if it did not want to induce high effort (Hint: this is "partial equilibrium", so the wage should be in terms of V_U which firms are taking as given). How does this wage compare to the one with high effort?*

4. *Again taking V_U as given, write down a condition determining when the firm would like to induce high effort. (Hint: in answering this question, recall that for a given total amount of efficiency units of labor, the firm can hire at the margin either one worker induced to exert high effort or $1/\alpha$ workers induced to exert no effort; you can use this observation to answer this question). How does the level of unemployment benefits affect this comparison? (Hint: an increase in z increases V_U).*

5. *Determine the steady state equilibrium levels of output and employment assuming that firms are happy with low effort (Hint: now solve for V_U; and also distinguish between the case in which there is full employment and the one with unemployment). Can the level of employment be lower than the high-effort equilibrium? Is the level of output higher? (Hint: think of the two extreme cases $\alpha \to 0$ and $\alpha \to 1$).*

6. *Now discuss the form of the equilibrium when firms choose between inducing high and low effort in general equilibrium. Can there exist an equilibrium in which some firms induce high effort and others induce low effort? (No need for derivations here, simply discuss the intuition). How would the level of unemployment benefits affect the form of the equilibrium? Briefly discuss the welfare implications of this extended model contrasting it with the baseline Shapiro-Stiglitz model (again no need for formal derivations here).*

Exercise 6 *The efficiency wage models we analyzed in the lecture were of the moral hazard variety (or effort-elicitation models). Another strand of the efficiency wage literature relies on adverse selection (type-elicitation).*

Suppose that there are N workers. ϕN of the workers are low type and have 1 efficiency unit of labor. $(1-\phi)N$ of the workers are high type and they have $\alpha > 1$ efficiency units of labor. The type of the worker is his private information and never observed by any other agent. High type workers have a reservation return u_h and low type workers have a reservation return $u_l < u_h$. There are M firms each with a decreasing returns to scale production function $F(H)$ where H is the efficiency units of labor.

1. *Draw the supply and demand curves for labor.*

2. *Assume that these two curves intersect at $w < u_h$. Show mathematically that it may be profitable for a firm to offer a wage $w = u_h$. Explain the intuition. Characterize diagrammatically the equilibrium in which all firms offer $w = u_h$. Find the unemployment rate of this economy. Is all of this "involuntary"? Why don't the employers cut wages.*

3. *The implicit assumption that you have used so far is that workers can apply to as many firms as they like. Now assume that each worker can only apply to one firm and choose which firm to apply after seeing the whole distribution of wage offers by firms. Show that starting from the allocation characterized (ii) where $w = u_h$ for all firms, there is a profitable deviation for a firm.*

4. *Can you guess the form of the equilibrium in this case where each worker can only apply to one firm?*

Exercise 7 *In his interviews with employers and employees, Truman Bewley finds that the key reason that is given for the unwillingness of firms to cut wages is "morale". He also finds that because of morale reasons, firms do not want to hire overqualified workers. Also firms that are in financial trouble face acute "morale" problems. In this problem you are asked to construct a model of morale.*

Time is discrete and all agents are risk-neutral with discount factor δ. Workers have disutility of work and unemployment benefit equal to 0. A firm consists of two workers. These two workers produce an output equal to y every period. However, in every period there is a probability q that one of the workers needs to exert an additional effort at cost e. If effort is exerted, output is again equal to y, otherwise output is equal to 0. Whether effort is required and if so whether it has been exerted or not is not observed by the firm. Finally, there is also a probability p that the firm will die in every period. Workers cannot be paid a negative wage, but each worker's wage can be conditioned on the history of the output of the firm.

1. *Characterize a low morale equilibrium in which workers receive a wage w when output is equal to y and in which they expect the other worker not to exert effort when there is need. Find the maximum profits for the firm in this case.*

2. *Characterize a high morale equilibrium in which workers expect the other worker to exert effort when needed as long as the history of output does not contain any 0 [thus workers are using trigger strategies]. What is the condition for the firm to prefer the high morale equilibrium?*

3. *Suppose the firm receives a negative shock such that p increases to $p\prime > p$. Show that the firm may actually have to pay higher wages in this case. Carefully explain the intuition. What does this imply about the behavior of wages over the cycle.*

4. *Suppose that the firm needs to hire a worker (say one of the workers quit unexpectedly). There are two candidates; A and B. A has the right qualifications and thus will produce y and will never quit. B is over-qualified. With B in place the firm will produce $y" > y$, but B also has a probability s of quitting and taking a better job in every period. Show that the firm may prefer to hire A.*