Political Economy of Mechanisms*

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Abstract

We study the provision of dynamic incentives to self-interested politicians who control the allocation of resources in the context of the standard neoclassical growth model. Citizens discipline politicians using elections. We show that the need to provide incentives to the politician in power creates political economy distortions in the structure of production, which resemble aggregate tax distortions. We provide conditions under which the political economy distortions persist or disappear in the long run. If the politicians are as patient as the citizens, the best subgame perfect equilibrium leads to an asymptotic allocation where the aggregate distortions arising from political economy disappear. In contrast, when politicians are less patient than the citizens, political economy distortions remain asymptotically and lead to positive aggregate labor and capital taxes.

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1 Introduction

We investigate how political economy affects dynamic resource allocation and taxation. As a first step in this direction, we study the dynamics of resource allocation in the electoral accountability model originally developed by Barro (1973) and Ferejohn (1986). In this class of models, politicians decide a range of policies and citizens can vote them out of office if dissatisfied with their performance. We combine this setup with the standard neoclassical growth model. The allocation of resources is indirectly determined by self-interested politicians who have access to a set of unrestricted tax instruments. In contrast to existing analyses of similar models, we model the economic decisions of citizens and the tax decisions of politicians without restricting attention to specific classes of tax policies (such as linear taxes) and we focus on subgame perfect equilibria that maximize citizens' ex ante utility, which we refer to as *the best sustainable mechanism*. Our focus on the best sustainable mechanism is motivated by our interest in understanding how the society might best avoid the distortions created by the presence of self-interested politicians and lack of commitment.[1] While the previous literature typically assumes stationary voting rules, we show that the best equilibrium is nonstationary and has qualitatively different implications than stationary equilibria (though it has a very simple structure and is renegotiation-proof).

Our results are closely related to and extend the literature on the dynamic principal-agent problem (see, among others, Harris and Holmstrom, 1982, Lazear, 1981, Ray, 2002). The most general formulation of dynamic principal-agent problems is provided in Ray (2002). Ray shows that the optimal provision of dynamic incentives induces backloading of payments to the agent. We show that similar backloading occurs in our economy (in the absence of capital) in the sense that politicians that remain in power for a long time are rewarded more. Conceptually, however, our focus is different from Ray's because we analyze political equilibria in a game between citizens and politicians, and we characterize equilibrium distortions and derive the conditions under which various different optimal tax structures are politically feasible. In addition, our technical results extend those in Ray (2002) in two significant directions. First, we allow the discount factors of citizens and politicians to differ. When politicians have a lower discount factor, backloading no longer applies and tax distortions remain even in the long run. Second, we analyze a dynamic economy with capital accumulation. The presence of capital introduces an additional state variable and implies that rewards to politicians are not

[1] Other equilibria will involve more distortions and will not necessarily answer the question of what the best feasible resource allocations are in the presence of political economy distortions.

necessarily backloaded even when they have greater discount factors than the citizens. These two differences are of substantive importance; politicians are often argued to be more short-sighted than the agents and the impact of political economy on intertemporal distortions (or on capital taxation) is one of the questions motivating our analysis.

Our paper is also related to and builds on the political economy literature.[2] The main difference between our approach and existing work in this literature is that we neither restrict citizens to stationary electoral policies nor impose exogenous restrictions on tax instruments. This generalized setup enables us to provide a tight characterization of the conditions under which political economy distortions persist or disappear in the long run. In contrast, as we show below, when attention is restricted to stationary strategies, these political economy distortions never disappear.[3]

2 Model

2.1 Preferences, Technology and Equilibrium

We consider an infinite horizon economy in discrete time, populated by a continuum 1 of identical individuals (citizens). Individual preferences at time $t = 0$ are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

where c denotes consumption and l is labor supply. We denote the set of citizens by I and use the subscript i to denote citizens. We impose the standard conditions on U:

Assumption 1 ***(utility)*** *$U(c, l)$ is twice continuously differentiable with partial derivatives denoted by U_C and U_L, strictly increasing in c, strictly decreasing in l and jointly concave in c and l. We adopt the normalization $U(0,0) = 0$. Moreover, $l \in \left[0, \bar{L}\right]$.*

The production side of the economy is described by the aggregate production function

$$Y_t = F(K_t, L_t), \tag{1}$$

which is defined inclusive of undepreciated capital (i.e., $F(K_t, L_t) \equiv \tilde{F}(K_t, L_t) + (1-\theta) K_t$ for some other production function $\tilde{F}(K, L)$ and for some depreciation rate $\theta \in (0,1)$).

[2] For example, Besley and Case (1995), Persson, Roland and Tabellini (2000), Acemoglu (2005) and Diermeier, Keane and Merlo (2006). See Persson and Tabellini (2000) and Besley (2006) for excellent overviews.

[3] Our work is also related to the growing literature on dynamic political economy. See, among others, Krusell and Rios-Rull (1999), Acemoglu and Robinson (2001, 2006), Hassler et al. (2005), and Battaglini and Coate (2006). In contrast to much of this literature, we focus on subgame perfect equilibria rather than Markovian equilibria. In this respect, our paper also builds on work on sustainable government policy in macro models, e.g., Chari and Kehoe (1990, 1993).

Assumption 2 ***(production structure)*** *F is strictly increasing and continuously differentiable in K and L with partial derivatives denoted by F_K and F_L, exhibits constant returns to scale, and satisfies $\lim_{L\to 0} F_L(K,L) = \infty$ for all $K \geq 0$ and $\lim_{K\to\infty} F_K(K,L) < 1$ for all $L \in [0,\bar{L}]$.*

The condition that $\lim_{K\to\infty} F_K(K,L) < 1$ together with $L \in [0,\bar{L}]$ implies that there is a maximum steady-state level of output that can be produced $\bar{Y} \in (0,\infty)$ uniquely defined by $\bar{Y} = F(\bar{Y},\bar{L})$. The condition that $\lim_{L\to 0} F_L(K,L) = \infty$ implies that in the absence of distortions there will be positive production.

The allocation of resources is delegated to a politician (ruler). The fundamental political dilemma faced by societies is to ensure that the body to which these powers have been delegated does not use them for its own interests. In the current model, this fundamental dilemma is partly resolved by the control of the politicians via elections.

We assume that there is a large number of potential (and identical) politicians, denoted by the set $\mathcal{I}$. Each politician's utility at time t is given by

$$\sum_{s=0}^{\infty} \delta^s v(x_{t+s}),$$

where x denotes the politician's consumption (rents) and $v : \mathbb{R}_+ \to \mathbb{R}$ is his instantaneous utility function. Notice also that the politician's discount factor, δ, is potentially different from that of the citizens, β. To simplify the analysis, we assume that potential politicians are distinct from the citizens and never engage in production and that once they are replaced they do not have access to capital markets (see footnote 7).

Assumption 3 ***(politician utility)*** *v is twice continuously differentiable, concave, and satisfies $v'(x) > 0$ for all $x \in \mathbb{R}_+$ and $v(0) = 0$. Moreover $\delta \in (0,1)$.*

The politician in power decides the allocation of resources (or equivalently decides a general set of taxes and transfers). The only restriction on the allocation of resources, in addition to $c_t \geq 0$ and $l_t \in [0,\bar{L}]$, comes from the *participation constraint* of the citizens, which requires that $U(c_t,l_t) \geq 0$ for each t.[4] We denote the three constraints $c_t \geq 0$, $l_t \in [0,\bar{L}]$ and $U(c_t,l_t) \geq 0$ by

$$(c_t,l_t) \in \Lambda \text{ for all } t. \tag{2}$$

[4]If the participation constraint $U(c_t,l_t) \geq 0$ is violated for some t, then citizens would supply zero labor at that date and secure utility $U(0,0) = 0$ without future negative repercussions (see below). However, note that this participation constraint only needs to be satisfied "along the equilibrium path"; the politician can *deviate* and induce an allocation that does not satisfy this constraint.

Since $U(c,l)$ is concave and continuous, Λ is closed and convex (and also nonempty). We use IntΛ to denote the interior of the set Λ, so that $(c_t, l_t) \in$IntΛ implies that $c_t > 0$, $l_t \in (0, \bar{L})$ and $U(c_t, l_t) > 0$.

We consider the following game. At each time t, the economy starts with a politician $\iota_t \in \mathcal{I}$ in power and a stock of capital inherited from the previous period, K_t. Then:

1. Citizens make labor supply decisions, denoted by $[l_{i,t}]_{i \in I}$, where $l_{i,t} \geq 0$. Output $F(K_t, L_t)$ is produced, where $L_t = \int_{i \in I} l_{i,t} di$.

2. The politician chooses the amount of rents $x_t \in \mathbb{R}_+$, a consumption function $\mathbf{c}_t : \mathbb{R}_+ \to \mathbb{R}_+$, which assigns a level of consumption for each level of (current) labor supply, and next period's capital stock $K_{t+1} \in \mathbb{R}_+$, subject to the constraint

$$K_{t+1} \leq F(K_t, L_t) - C_t - x_t,$$

where $C_t = \int_{i \in I} \mathbf{c}_t(l_{i,t}) \, di$ is aggregate consumption.[5] We sometimes use the shorthand $(x_t, \mathbf{c}_t, K_{t+1}) \in \Phi_t$ to denote a triple $(x_t, \mathbf{c}_t, K_{t+1})$ that is feasible for the politician.

3. Elections are held and citizens jointly decide whether to keep the politician or replace him with a new one, $\rho_t \in \{0, 1\}$, where $\rho_t = 1$ denotes replacement.

The important feature here is that even though individuals make their economic decisions independently, they make their political decisions—elections to replace the politician—jointly. This is natural since there is no conflict of interest among the citizens over the replacement decision. Joint political decisions can be achieved by a variety of procedures, including various voting schemes (e.g., Persson and Tabellini, 2000). Here we simply assume that the decision $\rho_t \in \{0, 1\}$ is taken by a randomly chosen citizen.[6]

We assume that at each date there is a public random variable z_t and all agents can condition their strategies on the history of this variable. This will enable us to convexify the value function of the citizens and is discussed in greater detail in the Appendix. Let

$$h^t \equiv \left(K_0, \iota_0, z_0, [l_{i,0}]_{i \in I}, x_0, \mathbf{c}_0, \rho_0, K_1, ...K_t, \iota_t, z_t, [l_{i,t}]_{i \in I}, x_t, \mathbf{c}_t, \rho_t, K_{t+1}\right)$$

denote the history of the game up to date t, and H^t be the set of all such histories. In the text, to simplify notation we suppress the conditioning on the history of z^t whenever this causes no confusion. A *subgame perfect equilibrium* (SPE) is given by labor supply decisions $\left[l^*_{i,t}\right]_{i \in I}$ at time t given history h^{t-1}, policy decisions $x^*_t, \mathbf{c}^*_t, K^*_{t+1}$ by the politician in power given h^{t-1} and $[l_{i,t}]_{i \in I}$, and electoral decisions by the citizens, ρ^*_t at

[5] One may wish to impose an additional constraint $x_t \leq \eta F(K_t, L_t)$ for some $\eta \in (0, 1)$, so that politician consumption cannot exceed an institutionally-imposed limit. This additional constraint does not affect our analysis and qualitative results, and is omitted to reduce notation.

[6] The same equilibrium arises from majoritarian or other types of elections over the replacement decision, since they will lead to the same worst punishment for the politician.

time t, given history h^{t-1} and $[l_{i,t}]_{i\in I}$, $x_t^*, \mathbf{c}_t^*, K_{t+1}^*$ that are best responses to each other for all histories.

We focus on SPE that maximize the utility of the citizens, which we refer to as the *best SPE*. Consider the following constrained optimization problem:

$$\textbf{MAX}: \max_{\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{3}$$

subject to an initial capital stock K_0, the resource constraint,

$$C_t + K_{t+1} + x_t \leq F(K_t, L_t) \text{ for all } t, \tag{4}$$

the sustainability constraint for the politician,

$$w_t \equiv \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v(F(K_t, L_t)) \text{ for all } t, \tag{5}$$

and the constraint (2). We have written this program using capital letters, since the consumption and labor supply levels refer both to individual and aggregate quantities. Notice also that in (5) we have defined w_t as the expected discounted utility of the politician at time t. This notation will be used in Theorem 1 below.

The sustainability constraint, (5), requires the equilibrium utility of the politician to be such that he does not wish to choose the maximum level of rents this period, $x_t = F(K_t, L_t)$, which would give him utility $v(F(K_t, L_t))$.[7] We refer to a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that is a solution to this problem as a *best sustainable mechanism* (since it implicitly defines a resource allocation mechanism).[8] The constraint, (5), is sufficient to ensure that the politician does not wish to deviate from the mechanism.

Proposition 1 *The allocation of resources in the best SPE (best sustainable mechanism) is identical to the solution of the maximization problem in* (MAX) *and involves no replacement of the initial politician along the equilibrium path.*

Proof. We first show that $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ that is a solution to (MAX) can be supported as a SPE with no politician replacement along the equilibrium path. Introduce

[7] Here we are using the assumption that the politician does not have access to capital markets. If he did, then after deviation he would not consume the entire amount $F(K_t, L_t)$ today, but would invest part of it in the capital market to achieve a smoother consumption profile. When the politician has access to capital markets, deviation from the implicitly-agreed mechanism becomes more attractive and thus (5) becomes more difficult to satisfy, though this does not affect any of our qualitative results.

[8] Conditioning on public histories, this sequence would be written as $\{C_t(z^t), L_t(z^t), K_{t+1}(z^t), x_t(z^t)\}_{t=0}^{\infty}$, since each element would be a function of the history of $z^t \equiv (z_0, ..., z_t)$.

the following notation: $h^t = \hat{h}^t$ if $(K_{s+1}(h^s), x_s(h^s)) = (\tilde{K}_{s+1}, \tilde{x}_s)$ and $\mathbf{c}_s(l_{i,s} \mid h^s) = \tilde{C}_s$ for $l_{i,s} = \tilde{L}_s$ and $\mathbf{c}_s(l_{i,s} \mid h^s) = 0$ for $l_{i,s} \neq \tilde{L}_s$ for all $s \leq t$. Consider the strategy profile ρ for the citizens such that $\rho(h^t) = 0$ if $h^t = \hat{h}^t$ and $\rho(h^t) = 1$ if $h^t \neq \hat{h}^t$. That is, citizens replace the politician unless the politician has always chosen a strategy inducing the allocation $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ in all previous periods. It is a best response for the politician to choose $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ after history h^t only if

$$\mathbb{E}\left[\sum_{s=0}^{\infty} \delta^s v\left(\tilde{x}_{t+s}\left(h^{t+s}\right)\right) \mid h^t\right] \geq \max_{\left(x_t', \mathbf{c}_t', K_{t+1}'\right) \in \Phi_t} \mathbb{E}\left[v\left(x_t'\right) + \delta v_t^c\left(K_{t+1}', \mathbf{c}_t', x_t'\right) \mid h^t\right]$$

where $v_t^c\left(x_t', \mathbf{c}_t', K_{t+1}'\right)$ is the politician's continuation value following a deviation to a feasible $\left(x_t', \mathbf{c}_t', K_{t+1}'\right)$. Under the candidate equilibrium strategy, $v^c = 0$ following any deviation, thus the best deviation for the politician is $x_t' = F(\tilde{K}_t, \tilde{L}_t)$, which gives (5). Consequently, (5) is sufficient for the politician not to deviate from $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$. Conversely, if some $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ violates (5) or the participation constraint in (2) after some history h^t, then this allocation cannot be sustained as a SPE (because either the politician or the citizens would deviate). Concavity of U combined with this observation establishes that no SPE can provide higher utility than $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$. Next suppose $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ that is a solution to (MAX) can be supported as a SPE with replacement of the initial politician. Now consider an alternative allocation $\{\tilde{C}_t', \tilde{L}_t', \tilde{K}_{t+1}', \tilde{x}_t'\}_{t=0}^{\infty}$ such that the initial politician is kept in power along the equilibrium path and receives exactly the same consumption sequence as the new politicians would have received after replacement. Since $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ satisfies (5) for the new politicians at all t, $\{\tilde{C}_t', \tilde{L}_t', \tilde{K}_{t+1}', \tilde{x}_t'\}_{t=0}^{\infty}$ satisfies (5) for all t for the initial politician. Moreover, since $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ must involve at least some positive consumption for the new politicians, $\{\tilde{C}_t', \tilde{L}_t', \tilde{K}_{t+1}', \tilde{x}_t'\}_{t=0}^{\infty}$ yields a higher $t = 0$ utility to the initial politician. Thus, x_0 can be reduced and C_0 can be increased without violating (5), so $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ cannot be a solution to (MAX). This proves that there is no replacement of the initial politician along the equilibrium path.

To complete the proof, we only need to show that citizens' strategy (in particular, $\rho(h^t) = 1$ if $h^t \neq \hat{h}^t$) is subgame perfect. This follows by considering the following continuation strategy for a politician: if $h^t \neq \hat{h}^t$, then $x_t = F(K_t, L_t)$ and $\mathbf{c}(l) = 0$ for all l, which is a best response to ρ, while replacement following $h^t \neq \hat{h}^t$ is a best response for the citizens given this strategy for the politician. ■

This proposition enables us to focus on the constrained maximization problem given in (MAX). Moreover, it implies that in the best SPE, the initial politician will be kept in power forever. This latter result follows since more effective incentives can be provided to the politician when he has a longer planning horizon (i.e., when he expects to remain in power for longer). Naturally, he is only kept in power along the equilibrium path—if he deviates from the implicitly-agreed mechanism, he will be replaced.[9]

For future reference, let us define an *undistorted* allocation as a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that maximizes (3) without the sustainability constraint (5) (for a given sequence of $\{x_t\}_{t=0}^{\infty}$). An undistorted allocation where $(C_t, L_t) \in \text{Int}\Lambda$ satisfies

$$F_L(K_t, L_t) U_C(C_t, L_t) = -U_L(C_t, L_t), \tag{6}$$

$$U_C(C_t, L_t) = \beta F_K(K_{t+1}, L_{t+1}) U_C(C_{t+1}, L_{t+1}). \tag{7}$$

We say that an allocation $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ features *downward labor distortions* at time t if the left-hand side of (6) is strictly greater than the right-hand side. Similarly, there are *downward intertemporal distortions* when the left-hand side of (7) is strictly less than the right-hand side. Downward distortions imply that there is less labor supply and less capital accumulation than in an undistorted allocation. We will interpret these distortions as corresponding to "aggregate tax distortions," since allocations that involve downward labor and intertemporal distortions can be decentralized by using linear labor and capital taxes.

2.2 The Best Sustainable Mechanism without Capital

Let us start with the economy without capital, so that instead of Assumption 2, we have $Y_t = L_t$. An allocation can now be represented by $\{C_t, L_t, x_t\}$ (thus dropping K_t). An undistorted allocation with $(C_t, L_t) \in \text{Int}\Lambda$ now satisfies $U_C(C_t, L_t) = -U_L(C_t, L_t)$.

We next introduce a sustainability assumption, which ensures that when the maximum amount of utility is given to the politician in every period, this is sufficient to satisfy the sustainability constraint (5). More formally:

Assumption 4 ***(sustainability)*** Let $(\tilde{C}, \tilde{L}) \in \arg\max_{(C,L)\in\Lambda} \{L - C\}$.
Then $v(\tilde{L} - \tilde{C})/(1 - \delta) > v(\tilde{L})$.

Let us also define *renegotiation-proof equilibria*, which we will use in the theorem. The issue of how renegotiation should be handled in dynamic games is not settled and there are many alternative notions in the literature (e.g., Fudenberg and Tirole, 1994).

[9] The continuation equilibrium in this proposition is subgame perfect but not "renegotiation-proof". We will define renegotiation-proofness below and show in Theorems 1 and 2 that renegotiation-proof continuation equilibria also support the same behavior. We do not incorporate renegotiation-proofness in Proposition 1 since it requires additional notation that will be introduced below.

Here, we adopt the simplest notion of renegotiation-proofness, which requires that the SPE play after any history h^t should not allow all active players to be made weakly better off (and some strictly better off). In this context, this implies that an alternative SPE should not make the citizens and the politician in power better off than the equilibrium.

Theorem 1 *Suppose that $Y_t = L_t$, that Assumptions 1, 3 and 4 hold and that $U_C(0,0) > U_L(0,0)$. Then in the best SPE (best sustainable mechanism), we have:*

1. there are downward labor distortions at $t = 0$.

2. when $\beta \leq \delta$, the values promised to the politician $\{w_t\}_{t=0}^{\infty}$ form a nondecreasing sequence and converge to some w^*. Moreover, $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ converges to some (C^*, L^*, x^*), which satisfies the no-distortion condition $U_C(C^*, L^*) = -U_L(C^*, L^*)$.

3. when $\beta > \delta$, then there are downward labor distortions even asymptotically.

The allocation described above can be supported as a renegotiation-proof SPE.

Proof. See the Appendix. ■

Part 1 of the theorem illustrates the additional distortion arising from the sustainability constraints. Intuitively, this distortion results because, as output increases, the sustainability constraint (5) requires that more be given to the politician in power and this increases the effective cost of production. The best SPE creates distortions so as to reduce the level of output and thus the rents that have to be paid to the politician.

Part 2 states that as long as $\beta \leq \delta$, the economy asymptotically converges to an equilibrium (C^*, L^*, x^*) where there are no aggregate distortions; even though there will be rents provided to the politician, these will be financed without introducing distortions. This result is important as it implies that in the long run there will be "efficient" provision of rents to politicians, with the necessary tax revenues raised without distortions (e.g., with lump-sum taxes in a decentralized allocation). This part of the theorem also shows that the (promised) rewards to the politician, given by the sequence $\{w_t\}_{t=0}^{\infty}$, are nondecreasing. Intuitively, current incentives to the politician are provided by both consumption in the current period, x_t, and by consumption in the future represented by the promised value, w_{t+1}. Future consumption by the politician not only relaxes the sustainability constraint in the future but does so in all prior periods as well. Thus, all else equal, optimal incentives for the politician should be backloaded. As discussed in the Introduction, this intuition for backloading is closely related to the results in the principal-agent literature (e.g., Ray, 2002).[10]

[10]Nevertheless, Theorem 1 here and Theorem 2 in the next subsection are not special cases of Ray's results and in fact extend them. First, these theorems cover the case with different discount factors. This is essential for our results regarding the long-run behavior of distortions. Second, with capital as an

Part 3 of the theorem states that if the politicians are less patient than the citizens, distortions will never disappear. Since in many realistic political economy models politicians are—or act as—more short-sighted than the citizens, this part of the theorem implies that in a number of important cases, political economy considerations will lead to additional distortions that will not disappear even asymptotically. Finally, Theorem 1 also shows that the best SPE can be supported as a renegotiation-proof equilibrium.

To provide an intuition for the proof of the theorem, let us represent the maximization problem in (MAX) recursively (for the special case without capital):

$$V(w) = \max_{(C,L)\in\Lambda,x,w^+} \left\{U(C,L) + \beta V(w^+)\right\} \tag{8}$$

subject to

$$C + x \leq L, \tag{9}$$

$$w = v(x) + \delta w^+, \tag{10}$$

$$v(x) + \delta w^+ \geq v(L). \tag{11}$$

Here $V(w)$ is the value of the citizens when they have promised value w to the politician and w^+ denotes next period's promised value. Constraint (9) imposes the resource constraint (4). Constraint (10) imposes promise keeping, incorporating the fact that the politician will not be replaced. It requires that the promised value w be equal to the sum of the current utility, $v(x)$, and the continuation utility, δw^+. Finally, constraint (11) is the recursive version of the sustainability constraint, (5). Let γ and $\psi \geq 0$ be the Lagrange multipliers on the constraints (10) and (11) respectively. Lemmas 2 and 4 in the Appendix imply that $V(w)$ is concave and differentiable. Furthermore, for the intuitive argument here, suppose that $(C,L) \in \text{Int}\Lambda$. The first-order condition with respect to w^+ and the envelope theorem then imply

$$\frac{\beta}{\delta}V'(w^+) = -\gamma - \psi = V'(w) - \psi. \tag{12}$$

Combining the first-order conditions for C and L gives

$$U_C(C,L) + U_L(C,L) = \psi v'(L). \tag{13}$$

Equation (13) makes it clear that aggregate distortions are related to the Lagrange multiplier on the sustainability constraint, ψ. Moreover, we must have $\psi > 0$ at $t = 0$,

additional state variable, we will have a dynamic game rather than a repeated game and the backloading result may not necessarily apply (see Theorem 2). Third, as the proof of Theorem 2 illustrates, the equilibrium nature of our problem necessitates an analysis of situations in which allocations converge to the boundary of the feasibility sets and thus requires a different strategy of proof. Finally and least importantly, Ray (2002) makes the opposite of Assumption 4 (or Assumption 4′ below).

otherwise the politician would receive $w_0 = 0$ initially, which together with (11) would imply $C_t = L_t = 0$ for all t. However, $C_t = L_t = 0$ for all t cannot be a solution when $\psi = 0$ at $t = 0$. Equation (13) then yields $U_C(C, L) + U_L(C, L) > 0$ at $t = 0$.

To obtain the intuition for the second part of Theorem 1, consider the case where $\beta = \delta$ (for the argument for $\beta < \delta$, see the Appendix). Then equation (12) implies

$$V'(w^+) = V'(w) - \psi \le V'(w). \tag{14}$$

Concavity of the value function $V(\cdot)$ then implies that $w^+ \ge w$, with $w^+ > w$ if $\psi > 0$, and $w^+ = w$ if $\psi = 0$. Therefore, the values promised to the politician form a nondecreasing sequence and converge to some w^* and (14) implies that ψ must converge to 0. This also implies that $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ converges to some (C^*, L^*, x^*), which satisfies (11) as stated in part 2 of Theorem 1.

This argument breaks down in part 3 of the theorem when $\delta < \beta$ because the politician does not value future rewards sufficiently and the sequence $\{w_t\}_{t=0}^{\infty}$ is not necessarily nondecreasing. In fact, (12) shows that if $\{w_t\}_{t=0}^{\infty}$ converges to some $\hat{w}$, then $\beta V'(\hat{w})/\delta = V'(\hat{w}) - \psi$, so that in this case ψ must be strictly positive and there must be asymptotic distortions (Lemmas 1 and 2 in the Appendix, together with the fact that $\{w_t\}_{t=0}^{\infty}$ is nondecreasing, imply that V' is negative in the relevant range of values).

2.3 The Best Sustainable Mechanism with Capital

We now extend Theorem 1 to an environment with capital, where the production function is given by Assumption 2. We first strengthen the sustainability assumption, Assumption 4. Let us define $\bar{C}$ and $\bar{K}$ such that

$$\bar{C} = \min\left\{C : \left(C, \bar{L}\right) \in \Lambda\right\} \text{ and } \bar{K} = \arg\max_{K \ge 0}\left\{F\left(K, \bar{L}\right) - K - \bar{C}\right\}. \tag{15}$$

Clearly $\bar{C}$ is uniquely defined (since $C \ge 0$ and Λ is closed). In view of this and Assumption 2, $\bar{K}$ is also uniquely defined.

Assumption 4′ (sustainability with capital) (1) $\delta v\left(F\left(\bar{K}, \bar{L}\right) - \bar{C} - \bar{K}\right)/(1-\delta) > v\left(F\left(\bar{K}, \bar{L}\right)\right)$; and (2) $\bar{C} + \bar{K} \le F\left(0, \bar{L}\right)$.

The first part of Assumption 4′ states that there exists a feasible allocation delivering sufficient utility to the politician so that the sustainability constraint (5) can be satisfied as a strict inequality.[11] A high discount factor δ is sufficient to ensure that this part of the assumption is satisfied. The second part of the assumption is a technical condition,

[11] This implies that the maximum utility to the politician can be provided without distortions.

which guarantees that the equilibrium allocation does not get stuck at some arbitrary capital level, and naturally requires that $F\left(0, \bar{L}\right) > 0$. Both parts of this assumption are used only in part 2 of the next theorem to characterize the equilibrium when the utility provided to a politician reaches the boundary of the set of feasible values.

Theorem 2 *Suppose that Assumptions 1-3 and 4′ hold. Then in the best SPE:*

1. there are downward labor distortions at some $t < \infty$ and downward intertemporal distortions at $t-1$ (provided that $t \geq 1$);

2. when $\beta \leq \delta$, the best sustainable mechanism $\{C_t, K_{t+1}, L_t, x_t\}_{t=0}^{\infty}$ converges to some (C^*, K^*, L^*, x^*). At this allocation, the labor and intertemporal distortions disappear asymptotically, i.e., (6) and (7) hold as $t \to \infty$;

3. when $\beta > \delta$, there are downward labor and intertemporal distortions, even asymptotically.

The allocation described above can be supported as a renegotiation-proof SPE.

Proof. See the Appendix. ∎

This theorem generalizes the results of Theorem 1 to an environment that is identical to the standard neoclassical growth model. The results are slightly weaker than in Theorem 1. In particular, there may not necessarily be distortions at the initial date, though such distortions will exist at some date. Perhaps more importantly, expected rewards for the politician are no longer increasing. In fact, it is straightforward to construct examples in which the initial capital stock is sufficiently high and these rewards are decreasing in the best SPE.[12] Also noteworthy is that when $\beta > \delta$, the best SPE not only generates labor distortions but also intertemporal distortions. These can be thought of as "aggregate capital taxes," since they create a wedge between the marginal product of capital and the ratio of marginal utilities of consumption. Therefore, this model generates a political economy rationale for long-run capital taxation.

2.4 Stationary Equilibria

We finally consider the best *stationary* SPE in the economy without capital. With stationary strategies, x_t has to be constant (conditional on the politician remaining in power).[13] The previous literature has typically focused on this type of stationary

[12] However, it can be shown that with capital, the second partial derivative of the value function of the citizens, $V_w(K, w)$, is nonincreasing (see the Appendix). This provides the appropriate notion of "backloading" in this generalized economy.

[13] A similar result can be stated for the economy with capital. In this case, politician consumption x would be a function of K, which complicates the analysis. The economy without capital allows us to emphasize the importance of non-stationary SPEs in a clearer fashion.

equilibria, in particular, assuming that individuals vote "retrospectively" according to some fixed threshold (see, for example, Persson and Tabellini, 2000, Chapter 4).

Proposition 2 *Consider the environment without capital in Theorem 1 and suppose that Assumptions 1, 3 and 4 hold and that* $U_C(0,0) > U_L(0,0)$. *Then, in the best stationary SPE distortions never disappear.*

Proof. Along a stationary equilibrium path, $x_t = x$ and $L_t = L$ so that

$$\frac{v(x)}{1-\delta} \geq v(L). \tag{16}$$

replaces the sustainability constraint (5). Constraint (16) must bind in all periods with $\psi > 0$, since otherwise the solution to the stationary equivalent of (MAX) would involve $x = 0$ and no distortions. The assumption that $U_C(0,0) > U_L(0,0)$ then implies that in this case $L > 0$, thus $x = 0$ would violate (16). Condition (13), which still applies in this case, then shows that there is a positive distortion on labor in all periods. ■

This proposition illustrates the role of nonstationary SPE in our analysis. Stationary equilibria do not allow the optimal provision of dynamic incentives to politicians and imply that political economy distortions never disappear, even when $\beta \leq \delta$.

3 Concluding Remarks

In this paper, we took a first step towards a political-economic analysis of dynamic resource allocation problems. We focused on economies in which allocation decisions are delegated to self-interested politicians and characterized the best equilibrium (from the viewpoint of the citizens). Political economy considerations lead to a new source of distortions in the allocation of resources (and thus to a new source of distortionary taxation) because of the necessity to satisfy the political sustainability constraints. We provided a full characterization of these distortions and their evolution over time. When politicians are as patient as, or more patient than, the citizens, these distortions disappear in the long run. The politician in power still receives rents but these rents are provided without additional distortions. In contrast, when politicians are less patient than the citizens, aggregate distortions remain positive even asymptotically. In this case, there will be asymptotic distortions that resemble positive labor and capital taxes.

The method of analysis presented here can be adapted to analyze the political economy of dynamic taxation in alternative environments. For example, in the dynamic Mirrlees taxation problem, individuals (citizens) also have private information and nonlinear tax schedules have to be incentive compatible to encourage the correct level of labor supply and effort (e.g., Mirrlees, 1971, Golosov, Kocherlakota and Tsyvinski, 2003). Acemoglu, Golosov and Tsyvinski (2006) show that in this environment the provision of incentives to politicians can be *separated* from the provision of incentives to individuals. This enables an analysis of this more general environment that is mathematically identical to the one presented here. The main result is that when politicians are as patient as (or more patient than) the citizens, the best SPE involves no distortions in addition to (and thus a structure of taxation similar to) those implied by the standard Mirrlees economy. Resources to compensate the politician are raised in a non-distortionary manner. Instead when politicians are less patient than the citizens, Mirrleesian taxes must be augmented by additional labor and capital distortions (taxes), even asymptotically.

Appendix: Technical Results and Omitted Proofs

Randomizations and the Properties of the Value Functions

We first formulate the characterization of the best SPE as a recursive program and ensure convexity by using randomizations. The recursive form of (3) is

$$V(K,w) = \max_{C,L,K^+,x,w^+} \left\{ U(C,L) + \beta V(K^+, w^+) \right\} \tag{A1}$$

subject to

$$C + x + K^+ \leq F(K,L), \tag{A2}$$

$$w = v(x) + \delta w^+, \tag{A3}$$

$$v(x) + \delta w^+ \geq v(F(K,L)), \tag{A4}$$

$$(C,L) \in \Lambda \text{ and } w^+ \in \mathbb{W}\left[K^+\right], \tag{A5}$$

where $\mathbb{W}[K^+]$ denotes the set of feasible values that can be provided to the politician starting with capital stock K^+. In particular, let us define the maximum utility that can be given to the politician when the capital stock is equal to K_t as

$$\bar{w}(K_t) \equiv \max_{\{C_{t+j}, L_{t+j}, K_{t+1+j}, x_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \delta^j v(x_{t+j}) \tag{A6}$$

subject to $(C_t, L_t) \in \Lambda$, (4) and (5) for all t. Evidently $\mathbb{W}[K] = [0, \bar{w}(K)]$.

Lemma 1 *The solution to the maximization problem (MAX) starting with the capital stock of K_0 is equivalent to the solution to the program (A1)-(A5) combined with a choice of initial promised value to the politician, w_0, such that $w_0 = \arg\max_{w \in \mathbb{W}[K_0]} V(K_0, w)$.*

Proof. The proof follows from Thomas and Worrall (1990). Clearly any solution to (A1)-(A5) gives a sustainable mechanism. Moreover, the ex ante utility for the citizens from any sustainable mechanism can be obtained as $V(K_0, w)$ from (A1)-(A5) by an argument analogous to the principle of optimality. It then follows that $V(K_0, w_0) = \max_{w \in \mathbb{W}[K_0]} V(K_0, w)$ gives the best sustainable mechanism. ■

The constraint (A14) in the program (A1)-(A5) is not convex and randomizations over the current consumption and the continuation value of the politician may improve the value of the program. This is the reason why we introduced the possibility of conditioning on the (payoff-irrelevant) public history $z^t \equiv (z_0, z_1, ..., z_t)$. Define $\mathbf{q} \equiv (C, L, K^+, x, w^+) \in \mathbb{R}^5$ and $\mathcal{C}(w) \equiv \{\mathbf{q} \in \mathbb{R}^5 :$ (A1)-(A5) are satisfied for given $w\}$ and let $\mathcal{Z}$ be the set of Borel subsets of $\mathcal{C}(w)$. Also define $\mathcal{P}(w)$ as the space of probability measures on $(\mathcal{C}(w), \mathcal{Z})$ and endow it with the weak topology. Incorporating randomization, we can write the recursive formulation as:

$$\textbf{Problem A1}: V(K, w) = \max_{\boldsymbol{\xi} \in \mathcal{P}(w)} \int [U(C, L) + \beta V(K^+, w^+)] \boldsymbol{\xi}(d\mathbf{q}) \tag{A7}$$

subject to

$$C + x + K^+ \leq F(K, L) \;\; \boldsymbol{\xi}\text{-almost-surely} \tag{A8}$$

$$v(x) + \delta w^+ \geq v(F(K, L)) \;\; \boldsymbol{\xi}\text{-almost-surely} \tag{A9}$$

$$w = \int [v(x) + \delta w^+] \boldsymbol{\xi}(d\mathbf{q}) \tag{A10}$$

$$(C, L) \in \Lambda \text{ and } w^+ \in \mathbb{W}\left[K^+\right] \;\; \boldsymbol{\xi}\text{-almost-surely.} \tag{A11}$$

The solution to this program will give stochastic sequences $\{x_t(z^t)\}_{t=0}^{\infty}$ and $\{w_t(z^t)\}_{t=0}^{\infty}$.

Lemma 2 *$V(K, w)$ is concave in w.*

Proof. Consider w_0 and w_1 and $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_1$ that are solutions to the maximization problem and let $w = (1-\alpha) w_0 + \alpha w_1$ and $\boldsymbol{\xi}_\alpha = (1-\alpha) \boldsymbol{\xi}_0 + \alpha \boldsymbol{\xi}_1$ for some $\alpha \in (0, 1)$. Constraints (A8) and (A9) are satisfied for both $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_1$, and therefore must be satisfied for $\boldsymbol{\xi}_\alpha$. Constraint (A10) is linear in $\boldsymbol{\xi}$, thus $\boldsymbol{\xi}_\alpha$ also satisfies this constraint. Since the objective function is linear in $\boldsymbol{\xi}_\alpha$, we also have $V(K, (1-\alpha) w_0 + \alpha w_1) \geq (1-\alpha) V(K, w_0) + \alpha V(K, w_1)$, establishing the concavity of V. ■

The next lemma shows that randomization using only two points is sufficient to achieve the maximum of (A7). The proof of this lemma is provided in Acemoglu, Golosov and Tsyvinski (2006) and is available in the Supplementary Appendix.

Lemma 3 *There exists $\boldsymbol{\xi} \in \mathcal{P}(w)$ that achieves $V(K, w)$ with randomization between two points, $(C_0, L_0, K_0^+, x_0, w_0^+)$ and $(C_1, L_1, K_1^+, x_1, w_1^+)$ with probabilities ξ_0 and $1 - \xi_0$.*

Lemma 3 implies that we can focus on randomizations between two points and can take aggregate public history to be of the form $z^t \in \{0,1\}^t$. Let us then denote the solutions for any w by $\left(C_i(w), L_i(w), K_i^+(w), x_i(w), w_i'(w), \xi_i(w) \text{ for } i \in \{0,1\}\right)$, naturally with $\xi_0(w) + \xi_1(w) = 1$. Rewrite Problem A1 in the equivalent form:

Problem A2: $$V(K,w) = \max_{\left\{\xi_i, K_i^+, C_i, L_i, x_i, w_i^+\right\}_{i=0,1}} \sum_{i=0,1} \xi_i \left[U(C_i, L_i) + \beta V(K_i^+, w_i^+)\right] \tag{A12}$$

subject to

$$C_i + x_i + K_i^+ \leq F(K, L_i) \text{ for } i = 0,1 \tag{A13}$$

$$v(x_i) + \delta w_i^+ \geq v(F(K, L_i)) \text{ for } i = 0,1 \tag{A14}$$

$$w = \sum_{i=0,1} \xi_i \left[v(x_i) + \delta w_i^+\right]. \tag{A15}$$

$$(C_i, L_i) \in \Lambda \text{ and } w_i^+ \in \mathbb{W}\left[K_i^+\right] \text{ for } i = 0,1. \tag{A16}$$

Lemma 4 *$V(K,w)$ is differentiable in w and K.*

The proof of this lemma is also provided in the Supplementary Appendix.

Proofs of Theorems 1 and 2

We start with the proof of Theorem 2, since some of the results in Theorem 1 will be obtained as corollaries.

Proof of Theorem 2. Since V is differentiable from Lemma 4 and concave from Lemma 2, the first-order conditions are necessary and sufficient for the maximization (A12). Assigning multipliers $\lambda_i \xi_i$ to the constraints (A13), $\psi_i \xi_i$ to (A14) and γ to (A15), and denoting the derivative of $V(K,w)$ with respect to w by $V_w(K,w)$, we have

$$\begin{aligned} \beta\xi_0 V_w\left(K_0^+, w_0^+\right) + \delta\psi_0\xi_0 + \delta\gamma\xi_0 &\leq 0 \\ \beta\xi_1 V_w\left(K_1^+, w_1^+\right) + \delta\psi_1\xi_1 + \delta\gamma\xi_1 &\leq 0, \end{aligned}$$

with both equations holding as equality for $w_i^+ \in \text{Int}\mathbb{W}\left[K_i^+\right]$. Therefore,

$$\frac{\beta}{\delta} V_w\left(K_i^+, w_i^+\right) \leq -\gamma - \psi_i, \tag{A17}$$

again with equality for $w_i^+ \in \text{Int}\mathbb{W}\left[K_i^+\right]$. Moreover, since V is differentiable,

$$V_w(K,w) \geq -\gamma \tag{A18}$$

again with equality for $w \in \mathrm{Int}\mathbb{W}\left[K_i^+\right]$. Combining the first-order conditions for C_i, L_i and K_i^+, we have that for $(C_i, L_i) \in \mathrm{Int}\Lambda$,

$$F_L(K, L_i)\, U_C(C_i, L_i) + U_L(C_i, L_i) = \psi_i v'(F(K, L_i))\, F_L(K, L_i) \text{ for } i = 0, 1, \quad \text{(A19)}$$

$$\beta \sum_{j \in \{0,1\}} \xi_j^+ F_K\left(K_i^+, L_j^+\right)\left[U_C\left(C_j^+, L_j^+\right) + \psi_j^+ v'\left(F\left(K_i^+, L_j^+\right)\right)\right] = U_C(C_i, L_i) \text{ for } i = 0, 1. \quad \text{(A20)}$$

Part 1: Suppose, to obtain a contradiction, that (A14) is slack for all t and $i = 0, 1$. Then the solution to (A12) involves $x_{i,t} = 0$ for all t and $i = 0, 1$, and thus $w_0 = 0$. Assumptions 1 and 2 imply that without any distortions $F(K_0, L_0) > 0$, thus the politician can deviate to $x_0 = F(K_0, L_0) > 0$ and increase his utility, yielding a contradiction. Therefore, (A14) must bind at some t and i with $\psi_{i,t} > 0$. Then (A19) implies that there will be downward labor distortions at that t, and (A20) implies that there will be downward intertemporal distortions at $t - 1$.

Part 2: Fix some $w \in \mathrm{Int}\mathbb{W}$. Since $\beta \leq \delta$ and $V_w(K, w) \leq 0$, (A17) implies

$$V_w\left(K_i^+, w_i^+\right) \leq -\gamma - \psi_i \text{ for } i = 0, 1.$$

Combining this with (A18) and $\psi_i \geq 0$ yields:

$$V_w(K, w) \geq V_w\left(K_i^+, w_i^+\right) \text{ for } i = 0, 1.$$

This implies that $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ is a nonincreasing (stochastic) sequence,[14] and necessarily converges on the extended real line. There are *three* cases to consider.

Case 1: $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ converges to some $V_w > -\infty$ and for all convergent subsequences $\{w_{t_n}, K_{t_n}\}_{t=0}^{\infty}$ of $\{w_t, K_t\}_{t=0}^{\infty}$, we have that K_{t_n} converges to some K_∞ and w_{t_n} converges to some $w_\infty \in \mathrm{Int}[0, \bar{w}(K_\infty)]$. This is only possible if the associated subsequence of multipliers $\{\psi_{t_n}\}_{t=0}^{\infty}$ converges to 0. Equations (A19) and (A20) then imply the desired result.

Case 2: $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ converges to some $V_w > -\infty$ and $\{w_t\}_{t=0}^{\infty}$ has a subsequence converging to $w_\infty \in \mathrm{Bd}[0, \bar{w}(K_\infty)]$ (where Bd denotes "boundary"). We now establish two lemmas that show that distortions also disappear in this case. Recall that $\bar{w}(K_t)$ denotes the maximum value that can be given to the politician starting with capital stock K_t. The next lemma states that if we reach the upper boundary of $\mathbb{W}[K_t] \equiv [0, \bar{w}(K_t)]$ at some t, we will always remain at the upper boundary of future $\mathbb{W}[K_t]$s.

[14] Here "nonincreasing" implies that every realization of V_w at time t is no less than its value at $t-1$. Throughout this proof, to reduce notation, we often suppress the stochastic nature of the sequences.

Lemma 5 *Let* $\{C^*_{t+j}, L^*_{t+j}, K^*_{t+1+j}, x^*_{t+j}\}_{t=0}^{\infty}$ *be the solution to the problem (A6) and recall that* $\bar{w}(K^*_t) = \sum_{j=0}^{\infty} \delta^j v(x^*_{t+j})$*. If* $w_{t'} = \bar{w}(K_{t'})$ *for some* t'*, then* $w_t = \bar{w}(K_t)$ *for all* $t \geq t'$*.*

Proof. Suppose to obtain a contradiction that the last statement is not true. Then there exists some feasible sequence $\{C_{t+j}, L_{t+j}, K_{t+1+j}, x_{t+j}\}_{j=0}^{\infty}$ and $K_{t+1+j^*} = K^*_{t+1+j^*}$ for some $j^* > 0$ such that $\sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s}) > \sum_{s=0}^{\infty} \delta^s v(x^*_{t+j^*+s})$. Now form the sequence $\left(\tilde{C}_{t+j}, \tilde{L}_{t+j}, \tilde{K}_{t+1+j}, \tilde{x}_{t+j}\right) = \left(C^*_{t+j}, L^*_{t+j}, K^*_{t+1+j}, x^*_{t+j}\right)$ for all $j < j^*$ and $\left(\tilde{C}_{t+j}, \tilde{L}_{t+j}, \tilde{K}_{t+1+j}, \tilde{x}_{t+j}\right) = (C_{t+j}, L_{t+j}, K_{t+1+j}, x_{t+j})$ for all $j \geq j^*$. This new sequence is feasible in view of the fact that $K_{t+1+j^*} = K^*_{t+1+j^*}$, and it gives value

$$\begin{aligned}
\tilde{w}(K_t) &= \sum_{s=0}^{j^*} \delta^s v(x^*_{t+j+s}) + \delta^{j^*} \sum_{s=0}^{\infty} \delta^s v(\tilde{x}_{t+j^*+s}) \\
&> \sum_{s=0}^{j^*} \delta^s v(x^*_{t+j+s}) + \delta^{j^*} \sum_{s=0}^{\infty} \delta^s v(x^*_{t+j^*+s}) = \bar{w}(K^*_t),
\end{aligned}$$

yielding a contradiction and establishing the lemma. ■

Lemma 6 *Suppose that Assumption 4′ holds and that* $w_{t'} = \bar{w}(K_{t'})$ *for some* $t' \geq 0$*. Then* $w_t > v(F(K_t, L_t))$ *for all* $t \geq t'$*.*

Proof. Suppose that $w_{t'} = \bar{w}(K_{t'})$ for some t'. Then, Lemma 5 implies that $w_t = \bar{w}(K_t)$ for all $t \geq t'$. Now to obtain a contradiction, suppose that at some $t \geq t'$ we have $w_t = v(F(K_t, L_t))$. By the second part of Assumption 4′, a feasible variation is as follows: $L_{t+s} = \bar{L}$ and $C_{t+s} = \bar{C}$ for all $s \geq 0$; $K_{t+s} = \bar{K}$ for all $s \geq 1$; and $x_t = F\left(K_t, \bar{L}\right) - \bar{C} - \bar{K}$ and $x_{t+s} = F\left(\bar{K}, \bar{L}\right) - \bar{C} - \bar{K}$ for all $s \geq 1$.

First suppose $F(K_t, L_t) \leq F\left(\bar{K}, \bar{L}\right)$. Then, this variation gives the politician value

$$\begin{aligned}
w' &= v\left(F\left(K_t, \bar{L}\right) - \bar{C} - \bar{K}\right) + \frac{\delta}{1-\delta} v\left(F\left(\bar{K}, \bar{L}\right) - \bar{C} - \bar{K}\right) \\
&\geq \frac{\delta}{1-\delta} v\left(F\left(\bar{K}, \bar{L}\right) - \bar{C} - \bar{K}\right) > v\left(F\left(\bar{K}, \bar{L}\right)\right),
\end{aligned}$$

where the first inequality exploits the first part of Assumption 4′ and the last inequality uses $F(K_t, L_t) \leq F\left(\bar{K}, \bar{L}\right)$. This yields the desired contradiction.

Next suppose that $F(K_t, L_t) > F\left(\bar{K}, \bar{L}\right)$ (which naturally implies that $K_t > \bar{K}$). Then the above variation gives the politician value

$$\begin{aligned}
w' &= v\left(F\left(K_t, \bar{L}\right) - \bar{C} - \bar{K}\right) + \frac{\delta}{1-\delta} v\left(F\left(\bar{K}, \bar{L}\right) - \bar{C} - \bar{K}\right) \\
&> v\left(F\left(K_t, \bar{L}\right) - F\left(\bar{K}, \bar{L}\right)\right) + v\left(F\left(\bar{K}, \bar{L}\right)\right) \\
&\geq v\left(F\left(K_t, \bar{L}\right)\right) \geq v\left(F\left(K_t, L_t\right)\right),
\end{aligned}$$

where the first inequality uses the first part of Assumption $4'$ and that $F\left(\bar{K},\bar{L}\right)>\bar{C}+\bar{K}$ (again from Assumption $4'$). The second inequality follows from the fact that for a concave function $f(x)\geq 0$, $f(x)\leq f(x-y)+f(y)$ for $y\leq x$, and the final inequality uses $L_t\leq\bar{L}$. The string of inequalities again leads to a contradiction, establishing that $w_t>v\left(F\left(K_t,L_t\right)\right)$ for all $t\geq t'$. ■

Consequently, even if $\{w_t\}_{t=0}^{\infty}$ converges to $w_\infty\in$Bd$[0,\bar{w}\left(K_\infty\right)]$ constraint (5) will ultimately become slack, so that $\psi_t\rightarrow 0$ and the desired result follows.

Case 3: $\{V_w\left(K_t,w_t\right)\}_{t=0}^{\infty}\rightarrow-\infty$ and there exists a subsequence of $\{w_t\}_{t=0}^{\infty}$ converging to some $w_\infty\in$Int$[0,\bar{w}\left(K_\infty\right)]$. This implies that either $\{\gamma_t\}_{t=0}^{\infty}\rightarrow\infty$ or $\{\psi_t\}_{t=0}^{\infty}\rightarrow\infty$. Then the first-order condition $v'\left(x_t\right)=\lambda_t/\left(\gamma_t+\psi_t\right)$ implies that either $x_t=\infty$ or $\lambda_t=\infty$. The former is impossible in view of the resource constraint (since $Y_t\leq\bar{Y}<\infty$ for all t). The latter would imply that $U_C\left(C_t,L_t\right)\rightarrow\infty$. Since U is concave, $U_C\rightarrow\infty$ is only possible when $C_t\rightarrow 0$. Since $(C,L)\in\Lambda$, this implies $L_t\rightarrow 0$, and from Assumption 2**,** $x_t\rightarrow 0$ and thus $w_t\rightarrow\bar{w}\left(K_\infty\right)$, which is in this case equal to 0. However, Lemma 6 implies that the best SPE with $w_t\rightarrow\bar{w}\left(K_\infty\right)$ cannot involve $w_t\rightarrow 0$, thus this case can be ruled out and this establishes Part 2.

Part 3: Suppose that $\beta>\delta$. If $\{V_w\left(K_t,w_t\right)\}_{t=0}^{\infty}$ converges to some V_w, then (A17) and (A18) imply that $\psi_t\rightarrow\psi_\infty>0$. Then equations (A19) and (A20) immediately imply that the asymptotic allocation is distorted downwards. Next, suppose that $\{V_w\left(K_t,w_t\right)\}_{t=0}^{\infty}$ does not converge. Nevertheless, it has a convergent subsequence (which may converge to $-\infty$, but this is ruled out by the same argument as in the previous part). (A17) and (A18) then imply that the multipliers associated with this subsequence satisfy $\psi_{i,t_n}\rightarrow\psi_{\infty,n}>0$. This establishes that $\limsup\left[F_L\left(K,L_i\right)U_C\left(C_i,L_i\right)+U_L\left(C_i,L_i\right)\right]>0$ and $\limsup\left[\beta F_K\left(K^+,L_i^+\right)U_C\left(C_i^+,L_i^+\right)-U_C\left(C,L\right)\right]>0$. Consequently, distortions do not disappear asymptotically.

To complete the proof, we need to establish renegotiation-proofness. For any initial level of capital K, let the equilibrium value of the (initial) politician be $w_0(K)$ and let the maximum value that can be promised to the politician be $\bar{w}\left(K\right)$. Recall the notation from the proof of Proposition 1, whereby $h^t\neq\hat{h}^t$ implies that the politician has deviated from the mechanism. Consider the following continuation equilibrium. If $\rho(h^t)=1$ and $h^t\neq\hat{h}^t$, then the continuation equilibrium is a solution to (MAX), with initial value for the next politician $w'=w_0\left(K\left(h^t\right)\right)$, where $K\left(h^t\right)$ is the capital stock after history h^t (that is, after the deviation if there is any). If $\rho(h^t)=1$ and $h^t=\hat{h}^t$, then the continuation equilibrium is a solution to (MAX), with initial value for the next politician given by $w'=\bar{w}\left(K\left(h^t\right)\right)\geq w_0\left(K\left(h^t\right)\right)$. Consequently, $\rho(h^t)=0$ following $h^t=\hat{h}^t$ is a best response for the citizens and involves the continuation play of a best SPE, thus the citizens and the politician in power cannot both be made better off. Similarly, $\rho(h^t) = 1$ following $h^t \neq \hat{h}^t$ is a best response and also involves the continuation play of a best SPE, thus the citizens and the (new) politician in power cannot both be made better off. This establishes that the best sustainable mechanism is supported as a renegotiation-proof SPE. ■

Proof of Theorem 1. The main results follow as corollaries of the equivalent results from Theorem 2. We thus only prove the three differences from that theorem. First, there are downward distortions at $t = 0$ (instead of at some $t < \infty$). Since there is no capital, if the sustainability constraint (5) were slack at $t = 0$, it would remain so at all future dates, implying that $x_t = 0$ for all t, and thus $w_0 = 0$. But $U_C(0,0) > U_L(0,0)$ implies that in the absence of distortions $L_0 > 0$, so that deviating and setting $x_0 = L_0 > 0$ would be a profitable deviation for the politician. This yields a contradictions and establishes that there must be downward labor distortions at $t = 0$.

Second, the sequence of $\{V_w(w_t)\}$s is nonincreasing. This, combined with the concavity of V, implies that $\{w_t\}_{t=0}^{\infty}$ is nondecreasing and thus converges to some $w^* \in [0, \infty]$.

Finally, Assumption 4 implies that $v(\tilde{L} - \tilde{C})/(1-\delta) > v(\tilde{L})$ and ensures that constraint (5) is slack when w_t converges to $w^* \leq v(\tilde{L} - \tilde{C})/(1-\delta)$. ■

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