

14.451: Introduction to Economic Growth

Problem Set 1

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Question 1: Consider a modified version of the continuous-time Solow growth model where the aggregate production function is

$$F(K, L, Z) = L^\beta K^\alpha Z^{1-\alpha-\beta},$$

where Z is land, available in fixed inelastic supply. Assume that $\alpha + \beta < 1$, capital depreciates at the rate δ , and there is an exogenous saving rate of s .

1. First suppose that there is no population growth. Find the steady-state capital-labor ratio in the steady-state output level. Prove that the steady state is unique and globally stable.
2. Now suppose that there is population growth at the rate n , that is, $\dot{L}/L = n$. What happens to the capital-labor ratio and output level as $t \rightarrow \infty$? What happens to returns to land and the wage rate as $t \rightarrow \infty$?
3. Would you expect the population growth rate n or the saving rate s to change over time in this economy? If so, how?

Question 2: Consider the Solow growth model with constant saving rate s and depreciation rate of capital equal to δ . Assume that population is constant and the aggregate production function is given by the constant returns to scale production function

$$F[A_K(t)K(t), A_L(t)L(t)]$$

where $\dot{A}_L(t)/A_L(t) = g_L > 0$ and $\dot{A}_K(t)/A_K(t) = g_K > 0$.

1. Suppose that F is Cobb-Douglas. Determine the balanced growth path (BGP) growth rate and the adjustment of the economy to the steady state.
2. Suppose that F is not Cobb-Douglas (even asymptotically). Prove that there does not exist a BGP. Explain why this is.
3. For the case in which F is not Cobb-Douglas, describe what happens to the capital-labor ratio and output per capita as $t \rightarrow \infty$.

Question 3: Consider the discrete-time Solow growth model with constant population growth at the rate n , no technological change and full depreciation, that is, $\delta = 1$. Assume that the saving rate is a function of the capital-labor ratio, thus given by $s(k)$.

1. Suppose that $f(k) = Ak$ and $s(k) = s_0 k^{-1} - 1$. Show that if $A = 1 + n$, then for any $k(0) \in (0, s_0)$, the economy immediately settles into an asymptotic cycle and continuously fluctuates between $k(0)$ and $s_0 - k(0)$.
2. Now consider more general continuous production function $f(k)$ and saving function $s(k)$, such that there exist $k_1, k_2 \in \mathbb{R}_+$ with $k_1 \neq k_2$ and

$$\begin{aligned} k_2 &= \frac{s(k_1) f(k_1)}{1+n} \\ k_1 &= \frac{s(k_2) f(k_2)}{1+n}. \end{aligned}$$

Show that when such (k_1, k_2) exist, there may also exist a stable steady state.

3. Prove that such cycles are not possible in the continuous-time Solow growth model for any continuous production function $f(k)$ and continuous $s(k)$. [Hint: consider the equivalent of Figure ?? above].
4. What does the result in parts 1-3 imply for the approximations of discrete time by continuous time suggested in Section ???
5. In light of your answer to part 5, what do you think of the cycles in parts 1 and 2?
6. Show that if $f(k)$ is nondecreasing in k and $s(k) = k$, cycles as in parts 1 and 2 are not possible in discrete-time either.

Question 4: Consider the basic Solow model in continuous time and suppose that $A(t) = A$, so that there is no technological progress of the usual kind. However, assume that the relationship between investment and capital accumulation is modified to

$$\dot{K}(t) = q(t) I(t) - \delta K(t),$$

where $[q(t)]_{t=0}^{\infty}$ is an exogenously given time-varying process. Intuitively, when $q(t)$ is high, the same investment expenditure translates into a greater increase in the capital stock. Therefore, we can think of $q(t)$ as the inverse of the relative prices of machinery to output. When $q(t)$ is high, machinery is relatively cheaper. Gordon (1990) documented that the relative prices of durable machinery has been declining relative to output throughout the postwar era. This is quite plausible, especially given our recent experience with the decline in the relative price of computer hardware and software. Thus we may want to suppose that $\dot{q}(t) > 0$.

1. Suppose that $\dot{q}(t)/q(t) = \gamma_K > 0$. Show that for a general production function, $F(K, L)$, there exists no steady-state equilibrium.
2. Now suppose that the production function is Cobb-Douglas, $F(K, L) = K^\alpha L^{1-\alpha}$, and characterize the unique steady-state equilibrium.
3. Show that this steady-state equilibrium does not satisfy the Kaldor fact of constant K/Y . Is this a problem? [Hint: how is “ K ” measured in practice? How is it measured in this model?].