Political Economy of Institutions and Development: 14.773
Problem Set 1

Due date: February, 29, 2008.

Question 1

1. Consider the example of a three-person three-policy society with preferences

$$\begin{array}{ll} 1 & a \succ b \succ c \\ 2 & b \succ c \succ a \\ 3 & c \succ b \succ a \end{array}$$

Voting is dynamic: first, there is a vote between a and b. Then, the winner goes against c, and the winner of this contest is the social choice. Find the subgame perfect Nash equilibrium voting strategy profiles in this two-stage game in "weakly undominated" (recall that each player's strategy has to specify how they will vote in the first round, and how they will vote in the second round as a function of the outcome the first round). [Hint: for "weakly undominated" strategies, first eliminate weakly dominated strategies in the last round, and then eliminate whatever is weakly dominated in the previous round, etc.].

2. Suppose a generalization whereby there are finite number of policies, $Q = \{q_1, q_2, ..., q_N\}$ and M agents (which you can take to be an odd number for simplicity). Voting takes $N - 1$ stages. In the first stage, there is a vote between q_1 and q_2. In the second stage, there is a vote between the winner of the first stage and q_3, until we have a final vote against q_N. The winner of the final vote is the policy choice of the society. Prove that if preferences of all agents are single peaked (with a unique bliss point for each), then the unique subgame perfect Nash equilibrium in "weakly undominated" implements the bliss point of the median voter.

3. *[For Bonus Points]* Why is "weakly undominated" in quotation marks? [Hint: can you construct other equilibria in parts 1 and 2, when you simply focus on weakly undominated strategies (that is, without doing the sequential elimination described in the Hint of part 1)?].

Question 2

Consider party competition in a society consisting of a continuum 1 of agents. The policy space is the $[0, 1]$ interval and assume that preferences of all agents are single peaked and political bliss points are uniformly distributed over this interval.

1. To start with, suppose that there are two parties, A and B. They both would like to maximize the probability of coming to power. The game involves both parties simultaneously announcing $q_A \in [0, 1]$ and $q_B \in [0, 1]$, and then voters voting for one of the two parties. The platform of the party with most votes gets implemented. Determine the equilibrium of this game. How would the result be different if the parties maximized their vote share rather than the probability of coming to power?

2. Now assume that there are three parties, simultaneously announcing their policies $q_A \in [0, 1]$, $q_B \in [0, 1]$, and $q_C \in [0, 1]$, and the platform of the party with most votes is implemented. Assume that parties maximize the probability of coming to power. Characterize all pure strategy equilibria.

3. Now assume that the three parties maximize their vote shares. Prove that there exists no pure strategy equilibrium. Characterize the mixed strategy equilibrium (Hint: assume the same symmetric probability distribution for two parties, and make sure that given these distributions, the third party is indifferent over all policies in the support of the distribution).

Question 3

Consider the following one-period economy populated by a mass 1 of agents. A fraction λ of these agents are capitalists, each owning capital k. The remainder have only human capital, with human capital distribution $F(h)$. Output is produced in competitive markets, with aggregate production function

$$Y = K^{1-\alpha} H^{\alpha},$$

where uppercase letters denote total supplies. Assume that factor markets are competitive and denote the market clearing rental price of capital by r and that of human capital by w.

1. Suppose that agents vote over a linear income tax, τ. Because of tax distortions, total tax revenue is

$$Tax = (\tau - v(\tau)) \left(\lambda rk + (1 - \lambda) w \int hdF(h) \right)$$

where $v(\tau)$ is strictly increasing and convex, with $v(0) = v'(0) = 0$ and $v'(1) = \infty$ (why are these conditions useful?). Tax revenues are redistributed lump sum. Find the ideal tax rate for each agent. Find conditions under which preferences are single peaked, and determine the equilibrium tax rate. How does the equilibrium tax rate change when k increases? How does it change when λ increases? Explain.

2. Suppose now that agents vote over capital and labor income taxes, τ_k and τ_h, with corresponding costs $v(\tau_k)$ and $v(\tau_h)$, so that tax revenues are

$$Tax = (\tau_k - v(\tau_k)) \lambda rk + (\tau_h - v(\tau_h)) (1 - \lambda) w \int hdF(h)$$

Determine ideal tax rates for each agent. Suppose that $\lambda < 1/2$. Does a voting equilibrium exist? Explain. How does it change when λ increases? Explain why this would be different from the case with only one tax instrument?

3. In this model with two taxes, now suppose that agents first vote over the capital income tax, and then taking the capital income tax as given, they vote on the labor income tax. Does a voting equilibrium exist? Explain. If an equilibrium exists, how does the equilibrium tax rate change when k increases? How does it change when λ increases?

Question 4

A society is a two party democracy with population normalized to 1, with political parties R and D competing to maximize their vote share. Parties compete by proposing a tax rate τ with proceeds distributed lump sum to each member of society. Taxing income introduces distortions, so the tax revenue, distributed lump-sum, is $(\tau - v(\tau))\bar{y}$, where $\bar{y}$ is average income in society and $v(0) = v'(0) = 0$ and $v'(1) = \infty$, and the government budget constraint is

$$T \leq (\tau - v(\tau))\,\bar{y}$$

The society is stratified into n groups. The size of each group varies, but members of the same group have the same income, denoted by y_j, but differing political ideology. Let the political leaning towards party R of individual i in group j be σ_j^i and the size of group j be α_j, with $\sum_{j=1}^{n} \alpha_j = 1$ and naturally $\sum_{j=1}^{n} \alpha_j y_j = \bar{y}$. Assume that σ_j^i has a symmetric distribution $\phi_j^{-1} F(x)$ with $\phi_j > 0$.

Assume that individuals of the society all share a common utility function

$$U_j^i(c_i, \sigma_j^i) = c_i + [\sigma_j^i + \delta] I_R$$

where I_R is an indicator for party R coming to power, and δ is a random popularity measure for party R, with distribution $G(\cdot)$.

1. First ignore the ideological leanings of each group and the relative popularity measure (i.e., $\sigma_j^i = \delta = 0$). Find the equilibrium in the party competition game and the tax rate announced by the two parties. Does a pure strategy equilibrium always exist?

2. Now characterize the equilibrium with the ideological leanings. Does a pure strategy equilibrium always exist?

3. Now assume the parties can offer group-specific transfers (instead of the lump sum redistribution) denoted by $\omega_j \geq 0$, so the government budget constraint thus becomes

$$\sum_{j=1}^{n} \alpha_j \omega_j \leq (\tau - v(\tau))\,\bar{y}$$

   Show, using an example, that there may exist no pure strategy equilibrium for the game when δ is known in advance to be 0. Determine conditions for an equilibrium to exist when δ is random with distribution G, and characterize such an equilibrium. Explain why a pure strategy equilibrium is more likely to exist when δ is random? Will the two parties necessarily offer the same policy platform?

4. Now fully characterize the equilibrium in this probablilistic voting model assuming that σ_j^i is uniform over $\left[-\phi_j^{-1}, \phi_j^{-1}\right]$ for all j and δ is uniform over $\left[-\psi^{-1}, \psi^{-1}\right]$.

Question 5

Consider a society consisting of δ_r rich agents, $\delta_m > \delta_r$ middle-class agents and $\delta_p > \delta_m + \delta_r$ poor agents. All individuals are infinitely lived in discrete time and maximize the net present discounted value of their lifetime income with discount factor $\beta \in (0,1)$. There are three political states: oligarchy (O), in which the rich agents are in power, limited franchise (L) in which the rich and the middle class vote, and full democracy in which all individuals are enfranchised (D). The society starts with oligarchy.

The political game is as follows: in each period, the median voter, of the prevailing political regime, decides a policy $\tau \in T \subset \mathbb{R}$, and also what the political regime should be tomorrow (from the set $\{O, L, D\}$). Each agent's income depends directly on the regime (for example, because different economic relationships are possible within different regimes), and on the policy τ. In particular, let $y_i(S, \tau)$ be the income of individual of class $i \in \{r, m, p\}$ in political state $S \in \{O, L, D\}$ when the policy is τ. Assume that the following are uniquely defined:

$$\begin{aligned} \tau^r &\equiv \arg\max_{\tau \in T} y_r(O, \tau) \\ \tau^m &\equiv \arg\max_{\tau \in T} y_m(L, \tau) \\ \tau^p &\equiv \arg\max_{\tau \in T} y_p(D, \tau). \end{aligned}$$

Individuals do not take any other action than the political actions described above.

1. Define a Markov Perfect Equilibrium (MPE) of this dynamic game.

2. Show that when $y_r(O, \tau^r) > \max\{y_r(L, \tau^m), y_r(D, \tau^p)\}$, the unique MPE involves the society remaining in oligarchy forever with policy τ^r. Explain the intuition for this result.

3. Suppose that $y_r(O, \tau^r) < y_r(L, \tau^m)$ and $y_m(D, \tau^p) < y_m(L, \tau^m)$. Show that in this case the unique MPE involves the society immediately switching to limited franchise and remaining there forever with policy τ^m. Interpret this result.

4. Suppose that $y_r(D, \tau^p) < y_r(O, \tau^r) < y_r(L, \tau^m)$, that $y_m(D, \tau^p) > y_m(L, \tau^m)$ and that $y_p(D, \tau^p) > \max\{y_p(O, \tau^r), y_p(L, \tau^m)\}$. Show that there exists β^* such that when $\beta < \beta^*$, the unique MPE involves the society becoming a limited franchise in the first period, then democracy in the following period, and remaining in democracy

with policy τ^p thereafter. When $\beta > \beta^*$, then the unique MPE involves a society remaining in oligarchy forever with policy τ^r. Explain why limited franchise cannot persist at the equilibrium regime in this case. Why is a higher discount factor making democracy less likely?