NASH MEETS RUBINSTEIN IN FINAL-OFFER ARBITRATION

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Abstract. Two players negotiate with alternating offers. If they fail to agree by a certain date, then an arbitrator makes the decision, by choosing one of the last two offers. The arbitrator maximizes the Nash's social-welfare function, and the discount factors are equal. As long as there are at least two rounds of negotiation, the backward induction outcome is as in the unique subgame-perfect equilibrium (SPE) of Rubinstein's infinite-horizon alternating-offer model: Player 1 offers his SPE offer in Rubinstein's model, and the other player accepts. Therefore, final-offer arbitration with Nash's social-welfare function terminates the game as quickly as possible without biasing the equilibrium outcome.

1. Introduction

Bilateral bargaining theory has two pillars: Nash's axiomatic bargaining solution, and Rubinstein's solution to the infinite horizon bargaining with alternating offers. Normalizing the payoffs in case of disagreement to zero, Nash (1950) has shown that if one wants to satisfy certain assumptions, then he must maximize the multiplication of the payoffs, which will be called the *Nash's social welfare function* hereafter. Rubinstein (1982) has shown that an infinite-horizon bargaining game with alternating offers has a unique subgame-perfect equilibrium. The two solutions are close when the discount rates are equal and close to 1 (Binmore, Rubinstein, and Wolinsky (1987)).

On the other hand, in formal negotiations, such as labor negotiations, some alternative dispute resolution mechanisms are used if the parties do not reach an agreement before a certain deadline. An important such mechanism is *final-offer arbitration*, introduced by Stevens (1966). If the parties cannot agree by a certain date, they have to go to a arbitrator, and the arbitrator has to choose between the last offers made by the parties. In the United States, final-offer arbitration is frequently used to resolve interest disputes in public-sector employment and to determine the salaries of professional baseball players.[1]

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[1]For more on final-offer arbitration, see a textbook, such as Murray, Rau and Sherman (1989).

It turns out that these three important pieces in dispute resolution share an interesting story. Suppose that in the final offer arbitration, the arbitrator is trying to maximize Nash's social-welfare function and the discount rates are equal. Then, the unique subgame-perfect equilibrium outcome in negotiation with final offer arbitration is precisely the unique subgame-perfect equilibrium outcome in Rubinstein's infinite-horizon bargaining game with alternating offers.

When the deadline is short, the mechanism used after the deadline usually has a large impact on the outcome of the negotiation in equilibrium. The result above shows that, regardless of how short the deadline is, as long as final-offer arbitration is used and the arbitrator maximizes Nash's social-welfare function, the equilibrium is as if there is no deadline. Such a mechanism does not introduce a bias in favor of any party.

In order to illustrate the main idea, consider a wage negotiation between an employer and a worker. Assume that both players are risk neutral and that they discount the future payoffs with δ in $(0,1)$. First the worker offers a wage w_0. If the employer accepts the offer, the worker gets w_0 and the employer gets $1-w_0$. If she rejects it, then she offers a wage w_1 in the next period. If the worker accepts w_1, the worker and the employer get δw_1 and $\delta(1-w_1)$, respectively. If he rejects the offer, then in the third period arbitrator sets the wage, choosing between w_0 and w_1. Maximizing Nash's social welfare function, the arbitrator chooses the offer that is closer to 1/2. Now suppose that the employer's offer w_1 is closer to 1/2 than w_0 is. The worker would not reject such an offer because the arbitrator would select w_1 in the next period anyway. If w_0 is closer to 1/2, then the worker accepts w_1 if $w_1 \geq \delta w_0$, as w_0 would be selected next day if he rejects w_1. Therefore, in equilibrium, the employer counteroffers $w_1(w_0) = \min\{\delta w_0, 1-w_0\}$. Notice that $w_1(w_0)$ is maximized at $w_0 = 1/(1+\delta)$, which is the equilibrium wage in Rubinstein's model. In the first period, if the worker offers $1/(1+\delta)$, then the employer would accept that offer because she is indifferent between paying $1/(1+\delta)$ in the first period and paying $w_1(1/(1+\delta)) = \delta/(1+\delta)$ in the next period. If the worker asks a higher wage, the employer rejects it because such a higher wage leaves less profit in the first period *and* leads to a lower wage in the next period, allowing a larger margin for profit the next period. Therefore, the worker offers $1/(1+\delta)$ in the first period, and the employer accepts it.[2]

[2]Since the counteroffer depends on the initial offer, the worker may prefer to make an initial offer that will be rejected if it leads to a better counteroffer. It turns out that this is not the case.

Notice that the dynamics of the counteroffers here does not resemble to the dynamics in usual the bargaining model. The counteroffer and the wage selected by the arbitrator are all determined by the initial offer. Nevertheless, the initial offer is as in the infinite-horizon version of the usual model. It turns out that this result remains true even when one allows arbitrary sets of payoff vectors and arbitrarily many rounds of negotiation.

In the next two sections, I will lay out the basic model and present some preliminary observations. I present the main result in Section 4. I extend the result to multi-period models of negotiation with endogenous and exogenous arbitration procedures in Section 5. Finally, in Section 6, I discuss the case with differing discount factors.

2. Model with Two Rounds of Negotiation

Consider the following perfect-information game, which will be called *the two-period final-offer arbitration model*. There are three players. The first two players, namely 1 and 2, are negotiators. The third player is an arbitrator, who will make the decision if the negotiators fail to agree. Formally, players 1 and 2 are to jointly choose a pair (x, y) from a convex, compact set X, where $X \subset \mathbb{R}^2_+$ is understood to be the set of all feasible expected utility pairs for players 1 and 2 after normalizing the disagreement payoffs to $(0,0) \in X$. There are three dates $t \in \{0,1,2\}$. If (x, y) is chosen at t, then the payoffs of players 1 and 2 are $\delta^t x$ and $\delta^t y$.

The timeline is as follows. At date $t = 0$, player 1 offers a pair $(x_0, y_0) \in X$, and player 2 accepts or rejects the offer. If he accepts the offer, then (x_0, y_0) is chosen, and the game ends; otherwise we proceed to $t = 1$. At $t = 1$, player 2 offers a pair $(x_1, y_1) \in X$, and player 1 decides whether to accept. If player 1 accepts the offer, (x_1, y_1) is chosen and the game ends. If this offer is also rejected, then the arbitrator makes the decision at $t = 2$.

This is a final-offer arbitration: the arbitrator has to choose either (x_0, y_0) or (x_1, y_1). I write

$$(x_2, y_2) \in \{(x_0, y_0), (x_1, y_1)\}$$

for the arbitrator's decision. The arbitrator's utility function at $t = 2$ is Nash's (1950) social welfare function:

$$u_A(x, y) = xy.$$

The arbitrator's preferences need to be of this form if his decision rule satisfies Nash's assumptions. There is, of course, no reason for Nash's assumptions to be satisfied in real life. I assume this specification here only because the result is about that case. The arbitrator's time preferences need not be specified because he moves only once.

I assume that X satisfies the standard assumptions in the bargaining literature: the function

$$f : x \mapsto \max \{y | (x, y) \in X\}$$

is concave, continuous, strictly decreasing with $f(0) > 0$ and $f(\bar{x}) = 0$ for some $\bar{x}$.

In the model above, the arbitrator uses the (final two) offers in the negotiation. Usually in real life, the parties submit those offers to the arbitrator, rather than arbitrator looking which offers have been made during the negotiation. In fact, in the Major Baseball League, the arbitrator is advised not to consider the offers made outside of the arbitration. This is not a concern as one can simply interpret (the last two rounds of) the negotiation in this model as part of the arbitration process in real life. This is because a party is allowed to accept the offer submitted to the arbitrator, rather than preparing a counteroffer or waiting for the arbitrator to make a decision. In fact, in the Major Baseball League, in 80% of the cases that are submitted to an arbitrator, the parties settle before the arbitrator makes a decision, which takes a month (see for example Wilson (1994)). Here, the crucial assumption is that the offers are submitted sequentially and the costs of delay is the same as in the negotiation. The assumption of sequentiality is sometimes natural because the party who files a case with an arbitrator has an incentive to submit his offer with the application and let the other party know what the offer is.

3. Preliminaries

This section describes the subgame-perfect equilibrium (henceforth SPE) in Rubinstein's alternating-offer model and the indifference curves of Nash's social welfare function. These will be needed to follow the arguments in the next section.

There is a unique SPE in Rubinstein's (1982) infinite-horizon, alternating-offer bargaining model. According to the SPE, player 1 offers a pair $\left(x^R, y^R\right)$ with $y^R = f\left(x^R\right)$, and player 2 accepts it. Rubinstein's solution is illustrated in Figure 1. Fixing the payoffs of player 1, scale down the payoffs of player 2 by δ, obtaining a curve that is defined by $y = \delta f(x)$. Now fixing the payoffs of player 2, scale down the payoffs of player 1, obtaining another curve, defined by $x = \delta f^{-1}(y)$. The unique intersection of these curves is $\left(\delta x^R, \delta f\left(\delta x^R\right)\right)$. When player 2 makes an offer, he offers $\left(\delta x^R, f\left(\delta x^R\right)\right)$, and it is accepted. The value of that outcome for player 2 in the previous period, when player

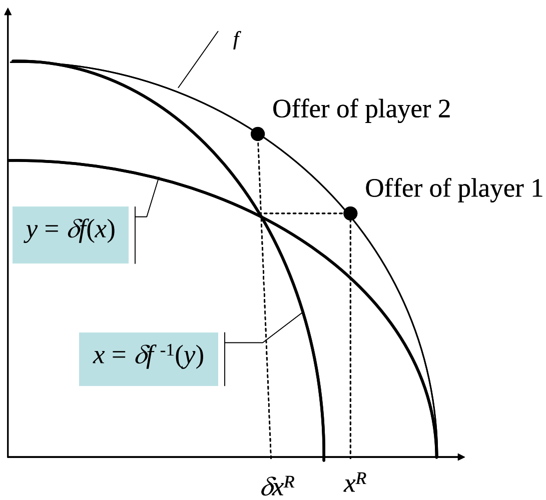

FIGURE 1. Rubinstein's solution

1 makes an offer, is $\delta f\left(\delta x^R\right)$. Player 1 gives that amount to player 2 and maximizes his payoff. He offers $\left(x^R, f\left(x^R\right)\right)$ where

$$f\left(x^R\right) = \delta f\left(\delta x^R\right), \tag{3.1}$$

making player 2 indifferent. This equation defines x^R.

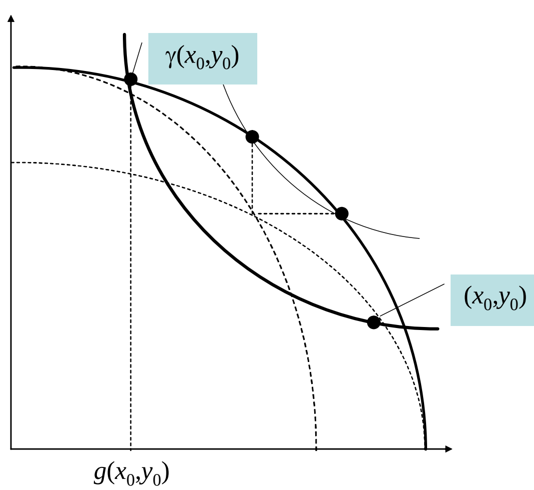

FIGURE 2. Indifference curves according to Nash's social-welfare function

The indifference curves with respect to the Nash's social welfare function are central for the final-offer arbitration model because they govern the arbitrator's behavior. Now, suppose that player 1 has offered (x_0, y_0) and will not accept any offer. What should player 2 offer at $t = 2$? Player 2 should offer the best point (x_1, y_1) with $x_1 y_1 \geq x_0 y_0$, for the arbitrator will chose the pair with the higher product. That is, he would offer $\gamma(x_0, y_0) \equiv (g(x_0, y_0), f(g(x_0, y_0)))$ with

$$g(x_0, y_0) = \min\{x | \exists (x, y) \in X : xy \geq x_0 y_0\}. \tag{3.2}$$

For an illustration, as in Figure 2, consider the indifference curve that contains (x_0, y_0). If (x_0, y_0) is the Nash bargaining solution, then the indifference curve is tangent to the Pareto frontier, with a unique intersection at (x_0, y_0). Otherwise, the indifference curve intersects the Pareto frontier twice. The pair $\gamma(x_0, y_0)$ is simply the intersection to the left. That is, $g(x_0, y_0)$ is the smaller of the two solutions to the equation

$$g(x_0, y_0) f(g(x_0, y_0)) = x_0 y_0. \tag{3.3}$$

The indifference curves of Nash's social welfare function and the Rubinstein's solution x^R have a special relation. As seen in Figure 2, the offer $\left(x^R, f\left(x^R\right)\right)$ of player 1 and the offer $\left(\delta x^R, f\left(\delta x^R\right)\right)$ of player 2 are on the same indifference curve. Indeed, multiplying both sides of (3.1) by x^R, one obtains

$$x^R f\left(x^R\right) = \delta x^R f\left(\delta x^R\right).$$

Since $\delta x^R < x^R$, this yields

$$g\left(x^R, f\left(x^R\right)\right) = \delta x^R. \tag{3.4}$$

This equality is another way to define x^R, as it has a unique solution, and it will be the crucial step in the proof of the main result.

4. Main Result

Proposition 1. *The outcome of the unique subgame-perfect equilibrium of the two-period final-offer arbitration model, which is given by backward induction, is the same as the outcome of the unique subgame-perfect equilibrium of Rubinstein's alternating-offer bargaining model: Player 1 offers*

$$(x_0, y_0) = \left(x^R, f\left(x^R\right)\right),$$

and player 2 accepts the offer.

Proof. I will now apply backward induction, by breaking the ties towards an equilibrium.

At $t=2$, given (x_0,y_0) and (x_1,y_1), the arbitrator will choose

$$(x_2,y_2)=\begin{cases}(x_1,y_1) & \text{if } x_1y_1\geq x_0y_0\\ (x_0,y_0) & \text{otherwise.}\end{cases}$$

(Since player 2 moves second, in equilibrium, arbitrator chooses (x_1,y_1) in case of indifference.)

Now consider the $t=1$ history in which player 2 offers (x_1,y_1) after rejecting (x_0,y_0). Suppose that $x_1y_1\geq x_0y_0$. Then, player 1 foresees that if he rejects the offer (x_1,y_1), in the next period, the arbitrator will choose the same decision (x_1,y_1). That is clearly worse than accepting the offer at $t=1$. Therefore, player 1 accepts the offer. Now suppose that $x_1y_1<x_0y_0$. Then, rejection leads to implementing (x_0,y_0) at $t=2$ with payoff δ^2x_0 to player 1, while acceptance leads to implementing (x_1,y_1) at $t=1$ with payoff δx_1 to player 1. Hence, player 1 will accept the offer if and only if

$$x_1\geq\delta x_0.$$

Now consider the node at which player 2 is to make an offer after rejecting (x_0,y_0). It is not a best reply for player 2 to make an offer that will be rejected: as we have just seen, such an offer will lead to the choice of (x_0,y_0) at $t=2$, while he can get $\gamma(x_0,y_0)$ accepted at $t=1$, leading to a higher payoff. Therefore, player 2 offers $(x_1^*(x_0,y_0),y_1^*(x_0,y_0))$ where

$$x_1^*(x_0,y_0)=\min\{g(x_0,y_0),\delta x_0\} \tag{4.1}$$

and

$$y_1^*(x_0,y_0)=f(x_1^*(x_0,y_0)). \tag{4.2}$$

Now consider $t=0$. I will show that player 1 will offer $(x^R,f(x^R))$, and the offer will be accepted, as desired. The proof consists of three steps.

Step 1: If player 1 offers $(x^R,f(x^R))$, it will be accepted, yielding the payoff of x^R for player 1.

Proof of Step 1: By (3.4), $x_1^*(x^R,f(x^R))=\delta x^R$ and hence $y_1^*(x_0,y_0)=f(\delta x^R)$. That is, if the offer $(x^R,f(x^R))$ is rejected, then $(\delta x^R,f(\delta x^R))$ will be implemented at $t=1$—precisely as in the SPE of Rubinstein's model. As we have seen there, player 2 is indifferent between accepting and rejecting the offer, and he will accept in equilibrium.

Step 2: Any offer (x_0,y_0) with $x_0>x^R$ is rejected and leads to a payoff strictly less than x^R for player 1.

Proof of Step 2: Note that (x_0, y_0) is in a lower indifference curve. Hence,

$$g(x_0, y_0) < g\left(x^R, f\left(x^R\right)\right) = \delta x^R < \delta x_0, \tag{4.3}$$

where the equality is by (3.4). Hence, by (4.1) and (4.2),

$$x_1^*(x_0, y_0) = g(x_0, y_0) \tag{4.4}$$

and

$$y_1^*(x_0, y_0) = f(g(x_0, y_0)) > f\left(\delta x^R\right), \tag{4.5}$$

where the inequality follows from (4.3) and the fact that f is strictly decreasing. Therefore,

$$\delta y_1^*(x_0, y_0) > \delta f\left(\delta x^R\right) = f\left(x^R\right) > f(x_0) \geq y_0,$$

where the equality is by (3.1), the next inequality is by the fact that f is strictly decreasing, and the last inequality is by definition of f. Since player 2 gets y_0 from acceptance and $\delta y_1^*(x_0, y_0)$ from rejection, he rejects the offer (x_0, y_0), as claimed. To see the second part of the claim, note that, by (4.1) and (4.3), the continuation payoff for player 1 after such an offer is

$$\delta x_1^*(x_0, y_0) = \delta g(x_0, y_0) < \delta^2 x^R.$$

Step 3: Any offer (x_0, y_0) with $x_0 < x^R$ leads to a payoff strictly less than x^R for player 1.

Proof of Step 3: If (x_0, y_0) is accepted, the payoff is $x_0 < x^R$. If (x_0, y_0) is rejected, then the continuation payoff, by (4.1), is

$$\delta x_1^*(x_0, y_0) \leq \delta^2 x_0 < x^R,$$

proving the claim.

Step 1 shows that the continuation value of offering $\left(x^R, f\left(x^R\right)\right)$ is x^R, which is strictly higher than the continuation value from any other offer, as established by Steps 2 and 3. Therefore, at $t = 0$, player 1 offers $\left(x^R, f\left(x^R\right)\right)$, and it is accepted by Step 1. □

Proposition 1 establishes that in a model of final offer arbitration in which the parties are allowed to accept the other parties' offers the subgame-perfect equilibrium outcome is the same as the one in Rubinstein's infinite-horizon bargaining model. As we will see later, this equivalence remains intact when the parties are allowed to negotiate before the arbitration. It is crucial for Proposition 1 that the offers are made sequentially. If the parties submit the offers simultaneously, as in the existing literature, the result is different. In that case, both parties offer the arbitrator's ideal payoff, which is the Nash's bargaining solution in our case (Crawford, 1979).

A basic intuition for the result is as follows. Suppose that player 1 can make only Pareto-efficient offers. Player 2 will accept an offer if and only if his payoff after the rejection, which depends on the offer that is being rejected, is less than or equal to his share according to the offer. Hence, when player 1 makes his offer, he has an incentive to minimize the continuation value of Player 2, which allows player 1 to get more out of the agreement. But the continuation value $y_1^*(x_0, f(x_0)) = f(x_1^*(x_0, f(x_0)))$ of player 2 is a decreasing function of the continuation value $x_1^*(x_0, f(x_0))$ of player 1. That is, player 1 has an incentive to maximize

$$x_1^*(x_0, f(x_0)) = \min\{g(x_0, f(x_0)), \delta x_0\}$$

over x_0. Clearly, as shown in Figure 3, δx_0 is increasing in x_0 and $g(x_0, f(x_0))$ is decreasing in x_0. Therefore, $x_1^*(x_0, f(x_0))$ is maximized at x_0 with

$$g(x_0, f(x_0)) = \delta x_0.$$

This is precisely the equation (3.4) that defines the Rubinstein's offer x^R. The equality is quite intuitive. If player 1 makes a greedy offer at $t = 0$, then player 2 would counter it with a slightly more equitable offer, which leads to a slightly higher social welfare according to Nash. (The counteroffer is accepted because it would have been selected by the arbitrator anyway.) On the other hand, if player 1 makes a less greedy offer and player 2 happens to reject it, then player 2's counteroffer would simply extract the gain from implementing the original offer at $t = 2$ rather than a period later. The equation equalizes these two incentives, which is also the property of Rubinstein's solution under equal discount rates.

5. Extensions to Multiperiod Negotiation

I have so far focused on a two period model of bargaining because in final-offer arbitration only the last two offers count, and the analysis of the bargaining differs from the standard bargaining only in the last two rounds. In that case the final-offer arbitration leads to the same outcome as Rubinstein's infinite-horizon model without arbitration. I will now show that this equivalence remains intact for arbitrary deadline and regardless of whether the arbitration is triggered exogenously (as in the compulsory arbitration) or endogenously (by one of the negotiators). I will start with the exogenous case.

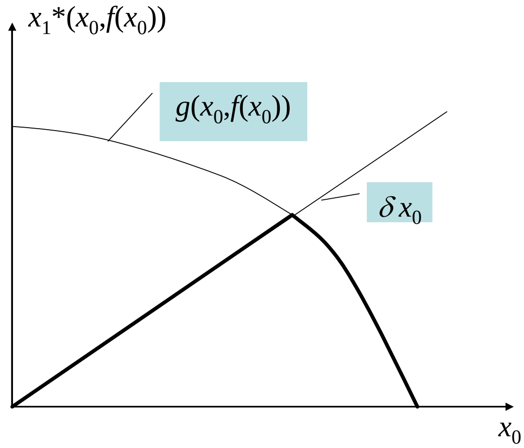

FIGURE 3. The payoff x_1^* of player 1 counteroffered by player 2, as a function of the initial offer x_0 of player 1.

Multi-period final-offer arbitration model: Suppose that the possible dates are $\{0, 1, 2, \ldots, T-1, T\}$, where $T \geq 2$. Players 1 and 2 negotiate and they go to final offer arbitration at date T if they have not agreed by then. At each $t < T$, one of the players—player 1 at even dates and player 2 at odd dates—makes and offer, and the other player decides whether to accept the offer, ending the game, or reject it and go to the next period. The payoffs and the set X are as before.

Proposition 2. *The outcome of the unique subgame-perfect equilibrium of the multi-period final-offer arbitration model, which is given by backward induction, is the same as the outcome of the unique subgame-perfect equilibrium of Rubinstein's infinite-horizon alternating-offer bargaining model: Player 1 offers*

$$(x_0, y_0) = \left(x^R, f\left(x^R\right)\right),$$

and player 2 accepts the offer.

Proof. By Proposition 1, at $T-2$, the proposer offers his SPE offer in Rubinstein's model and the other player accepts it—independent off all the previous offers. But then, in backward induction, the reduced game for dates $\{0, 1, 2, \ldots, T-3\}$ is the alternating-offer bargaining model with the same termination value as that of the SPE in Rubinstein's model. Then, backward induction leads to the same behavioral strategies at dates $\{0, 1, 2, \ldots, T-3\}$ as in the Rubinstein's model. In particular, at $t = 0$, player 1 offers $\left(x^R, f\left(x^R\right)\right)$, and player 2 accepts it. □

In the multiperiod model above, the arbitration is triggered exogenously at a fixed date. Alternatively, arbitration may be triggered if one of the parties endogenously files a case with the arbitrator. I will next extend the result to this case, where I will put the final offers in the arbitration process.[3]

Endogenous final-offer arbitration model: Suppose that the possible dates are all natural numbers $\{0, 1, 2, \ldots\}$. Players 1 and 2 negotiate. At each t, one of the players—player 1 at even dates and player 2 at odd dates—makes and offer, and the other player decides whether to

- accept the offer, ending the game, or
- reject it and file for an arbitration in the next period, or
- reject it and remain in the negotiation.

In the arbitration, the party who files the case submits an offer to the arbitrator. The other party either accepts it or submits a counteroffer to the arbitrator (rejecting the former offer). The party who files the case, may accept the counteroffer or rejects it, in which case the arbitrator selects between the two offers and the game ends. The payoffs and the set X are as before.

Note that the only difference between filing a case and remaining in the negotiation is that in the latter case the party also triggers an arbitration procedure with his offer, ending the game after two more periods. Moreover, as in the usual bargaining models with outside options, only the responder has an option to file a case, presumably the proposer could not file a case when he has an offer that is not rejected yet.

The main result extends to this case intact:

Proposition 3. *The unique subgame-perfect equilibrium outcome of the endogenous final-offer arbitration model is the same as the unique subgame-perfect equilibrium outcome of Rubinstein's infinite-horizon alternating-offer bargaining model: Player 1 offers*

$$(x_0, y_0) = \left(x^R, f\left(x^R\right)\right),$$

and player 2 accepts the offer.

Proof. Since we can take the arbitration procedure as the outside option of the responder, the result follows from the result of Binmore, Shaked and Sutton (1989). Now, by Proposition 1, the value of going into final-offer arbitration, which is the responder's outside option, is precisely the same as remaining in the negotiation. Therefore, the

[3]This extension came out of a discussion with Alp Simsek.

unique subgame-perfect equilibrium outcome is the same as the one in Rubinstein's model with no outside option. □

Notice that in the endogenous final-offer arbitration model, there are multiple subgame-perfect equilibria, as the responder is indifferent between filing a case with the arbitrator or remaining in the negotiation. They all lead to the same outcome nonetheless. Notice also that the result would not change if there were also mandatory final-offer arbitration starting at some $T \geq 2$ (where the offers are submitted to the arbitrator sequentially).

6. Differing Discount rates

The assumption that the discount rates are equal is needed for the result. For an illustration, consider the example in the introduction, and assume that worker and employer discount the future payoffs with δ_1 and δ_2, respectively. Then,

$$w_1(w_0) = \min\{\delta_1 w_0, 1 - w_0\}.$$

Hence, the employer accepts the initial offer iff

$$1 - w_0 \geq \delta_2 (1 - w_1(w_0)) = \delta_2 \max\{1 - \delta_1 w_0, w_0\}.$$

That is, she accepts the offer w_0 iff

$$w_0 \leq \min\left\{\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{1}{1+\delta_2}\right\}.$$

The first entry on the right hand side is Rubinstein's solution with differing discount factors, and the second entry is the solution when both use δ_2 as the discount rate. When $\delta_2 \geq \delta_1$, the result will be as in Rubinstein's model with different discount rates. In that case, the employer accepts the initial offer iff

$$w_0 \leq \frac{1-\delta_2}{1-\delta_1\delta_2}.$$

The worker either offers Rubinstein's solution, which will be accepted, or goes for the highest counteroffer, which is obtained at $w_0 = 1/(1+\delta_1)$, yielding the payoff $\delta_1^2/(1+\delta_1)$. The latter payoff is lower, and he offers Rubinstein's solution. On the other hand, when $\delta_2 < \delta_1$, the employer accepts w_0 iff

$$w_0 \leq \frac{1}{1+\delta_2}.$$

Now, the worker offers $1/(1+\delta_2)$, which will be accepted. (The best rejected offer, which is $1/(1+\delta_1)$, leads to a clearly lower payoff as in the previous case.) This is clearly different from Rubinstein's solution with differing discount rates. The outcome is equal to Rubinstein's solution with the worker's discount factor is taken as $\min\{\delta_1, \delta_2\}$, which is also true in general.

This assumption is not as restrictive as it may appear. It is well-known that one can choose to model the players' time preferences using the same discount factor and appropriate utility function for each player (see for example Binmore, Rubinstein, and Wolinsky (1986)). The result applies under the representation with equal discount factors.

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