14.773 Political Economy of Institutions and Development.
Lecture 14. Political Agency and Electoral Control.

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Introduction

- Voting as a way of resolving conflicts over policies.
- But in representative democracies, policies entrusted to politicians.
- How to ensure that politicians implement policies consistent with voter preferences?
- Barro-Ferejohn model: elections as a politician control device.
- Voters vote politicians who do not "perform" out of office.
- Loosely speaking, politicians as "agents" and voters as "principals".

Static Model with Full Information

- Suppose that there is some parameter θ, capturing the cost of supplying public goods.
- The government budget constraint is

$$\tau \bar{y} = \theta g + r \tag{1}$$

- Here $\bar{y}$ is average income, τ is the tax rate, g is the level of public good provision, r is the rents captured by the politician or political party in power.
- θ is a random variable with distribution $F(\cdot)$, whereby high values of θ correspondent to high costs of providing the public good.

Static Model (continued)

- Let the indirect utility function of the representative (or decisive) voter be

$$U(g(\theta), r(\theta), \theta) = \bar{y} - \theta g(\theta) - r(\theta) + H(g(\theta)).$$

- Here the function H captures their valuation of the public good.
- Suppose that H is increasing and strictly concave.
- Let $g^*(\theta)$ denote the preferred level of public good when the cost is θ, so that

$$H'(g^*(\theta)) = \theta.$$

- Since preferences are quasi-linear preferences, $g^*(\theta)$ independent of r, and consequently $g^*(\theta)$ is increasing.

Static Model (continued)

- We start with an incumbent politician with the following utility function
$$\mathbb{E}(v_I) = r + p_I R.$$
- Here R is the total rent from continuing to stay in power and p_I is the probability that the voters will keep the incumbent in power.
- Since we are in the full information, voters observe θ.
- Suppose that voters coordinate on (and commit to) the following *retrospective* voting strategy:
$$p_I = \begin{cases} 1 & \text{if } U(g(\theta), r(\theta), \theta) \geq \varpi(\theta) \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$
- In other words, the voters will keep the incumbent in power, $p_I = 1$, if he delivers utility greater than $\varpi(\theta)$ to them.
- In this static model, retrospective voting is natural.

Static Model (continued)

- Given (2), the politician solves the following program:

$$\max r(\theta) + p_I R$$

subject to

$$p_I = 1 \text{ only if } \bar{y} - \theta g(\theta) - r(\theta) + H(g(\theta)) \geq \varpi(\theta). \tag{3}$$

- Let us first look for a solution in which $p_I = 1$.
- It is then clear that the politician will never leave the constraint slack.
- Moreover, since $g(\theta)$ does not appear in the objective function, it will be chosen so as to maximize the choice set of the politician, i.e., increase the LHS of (3) by setting $g(\theta) = g^*(\theta)$.
- Therefore:

$$r(\theta) = \bar{y} - \theta g^*(\theta) - \varpi(\theta) + H(g^*(\theta)).$$

Static Model (continued)

- The next step is to check that the politician indeed prefers $p_I = 1$. With $p_I = 1$, he receives

$$\bar{y} - \theta g^*(\theta) - \varpi(\theta) + H(g^*(\theta)) + R.$$

- In contrast, if he chooses $p_I = 0$, then he sets the highest possible level of r, $r = \bar{y}$, and obtains $\bar{y}$.
- Thus for the politician to behave according to the wishes of the voters, we need

$$\gamma\bar{y} - \theta g^*(\theta) - \varpi(\theta) + H(g^*(\theta)) + R \geq \bar{y} \tag{4}$$

Static Model (continued)

- Now we have to think about how $\varpi(\theta)$ is determined.
- Trivially, there is a coordination problem, so various different levels of $\varpi(\theta)$ could emerge.
- Let us ignore this coordination problem and imagine that voters can coordinate on the best $\varpi(\theta)$.
- This will be such that (4) holds as equality:

$$\varpi(\theta) = [H(g^*(\theta)) - \theta g^*(\theta)] + R,$$

as long as $\bar{y} \geq R$ (if this inequality does not hold, then (4) cannot hold as an equality—assuming that there cannot be negative rents).

- This voting rule (with full information) then yields rents equal to:

$$r^* = \bar{y} - R$$

- *Important:* the incumbent politician is receiving rents despite the fact that there is electoral control over him. This is because of the advantage of incumbency, in other words, his ability to control policy.

Dynamic Model with Full Information

- The only modification now is that the objective function of the politician at time $t = 0$ is changed to

$$\mathbb{E}(v_I) = \sum_{t=0}^{\infty} \beta^t r_t,$$

with the convention that if $p_t = 0$, so that he is not elected at time t, then $r_{t+k} = 0$ for all $k > 0$.

- The budget constraint is

$$\tau_t \bar{y} = \theta_t g_t + r_t$$

(in other words, there needs to be budget balance in every period).

- Let us look for a *stationary* (and Markov) voting rule, where voters only condition on current performance in deciding whether to elect the politician. *Important and restrictive...*
- It also implies that voting rules must be *retrospective.*

Dynamic Model (continued)

- With the restriction to stationary strategies, the voting rule must take the form

$$p_t = \begin{cases} 1 & \text{if } U(g_t(\theta_t), r_t(\theta_t), \theta_t) \geq \varpi(\theta_t) \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

- How is $\varpi(\theta_t)$ determined?
- The "reverse logic" of the static model.
- In any given period, the politician can choose $r_t = \bar{y}$, so the problem is to convince him not to do so.
- Let $R_{t+1} = \sum_{t=1}^{\infty} \beta^{t-1} r_t$ be the continuation value.
- Then, we have an incentive compatibility constraint for the politician of the form:

$$r_t + \beta R_{t+1} \geq \bar{y}$$

- If this condition is violated, the politician will steal everything.

Dynamic Model (continued)

- The objective of the citizens is to make sure that this constraint is satisfied while giving the least amount of rents to the politician.
- The best way of doing so is to have the politician remain in power with probability one as long as he follows the prescribed policy.
- Let r be the per period rent, which is constant by the stationarity assumption.
- Then

$$R_{t+1} = \frac{r}{1-\beta}$$

- The stationary equilibrium therefore involves

$$r = (1-\beta)\,\bar{y}.$$

Dynamic Model (continued)

- This then implies that voters set the retrospective voting rule (5) with

$$\varpi(\theta) = \beta\bar{y} - \theta g^*(\theta) + H(g^*(\theta)),$$

which will deliver exactly $r(\theta) = (1-\beta)\,\bar{y}$ to the politician in all states.

- The dynamic model, therefore, highlights more clearly that the source of the power of the politician is his ability to choose the current policy, and voters have to adjust their voting rule so as to leave him sufficient rents.

Political Agency with Asymmetric Information

- Now suppose that voters do not observe θ, and let's understand the structure of the static model in this case.
- Since voters cannot observe θ, the only static voting policy they can choose is of the form

$$p_I = 1 \quad \text{iff} \quad U(g_t(\theta_t), r_t(\theta_t), \theta_t) \geq \varpi.$$

  (here the optimality of voting rules is much more complicated).
- Now let us look at the politician's strategy. The same argument as above implies that if the politician decides to satisfy the voters, he will choose

$$r(\theta) = \bar{y} - \theta g^*(\theta) - \varpi + H(g^*(\theta)).$$

- Note that the choice of g will reveal the underlying state, but voters do not condition the election rule on this. Why is that?

Agency with Asymmetric Information (continued)

- In this case, the politician receives

$$\bar{y} - \theta g^*(\theta) - \varpi + H(g^*(\theta)) + R.$$

- It is also satisfying the constraint that voters have put as an equality, so

$$U(g_t(\theta_t), r_t(\theta_t), \theta_t) = \varpi.$$

- In contrast, as before, if he chooses $p_I = 0$, then he sets the highest possible level of r, $r = \bar{y}$, and obtains $\bar{y}$.
- Thus, the politician will get reelected if

$$H(g^*(\theta)) - \theta g^*(\theta) - \varpi + R \geq 0$$

- Define θ^* such that

$$H(g^*(\theta^*)) - \theta^* g^*(\theta^*) - \varpi + R = 0 \tag{6}$$

Agency with Asymmetric Information (continued)

- Then it is clear that voters' utility will be

$$U(g_t(\theta_t), r_t(\theta_t), \theta_t) = \begin{cases} \varpi & \text{for } \theta \leq \theta^* \\ 0 & \theta > \theta^* \end{cases}$$

- Therefore, their expected utility is

$$\mathbb{E}(U) = \int_{\underline{\theta}}^{\theta^*} \varpi dF(\theta) + \int_{\theta^*}^{\overline{\theta}} 0 \cdot dF(\theta) = F(\theta^*)\varpi \tag{7}$$

- Let us next look at the relationship between ϖ and θ from (6).
- The implicit function theorem gives:

$$\frac{d\theta^*}{d\varpi} = \frac{1}{\left[H'(g^*(\theta^*)) - \theta^*\right] (g^*)'(\theta^*) - g^*(\theta^*)} = -\frac{1}{g^*(\theta^*)} < 0,$$

  where the second equality uses the envelope theorem.
- This highlights the major trade-off that voters face: if they set a high level of ϖ, then their utility is higher when the politician behaves well, but there is a higher probability that he will not do so.

Agency with Asymmetric Information (continued)

- Now consider the maximization of with welfare respect to ϖ:

$$\max_{\varpi} \mathbb{E}(U) = F(\theta^*)\varpi$$

- The first-order condition for this is

$$\begin{aligned} F(\theta^*) - f(\theta^*)\varpi\frac{d\theta^*}{d\varpi} &= 0 \\ F(\theta^*) - \frac{f(\theta^*)\varpi}{g^*(\theta^*)} &= 0 \end{aligned}$$

- The interpretation is as follows: a unit increase in ϖ benefits the citizens by an amount $F(\theta^*)$, since this is the probability with which they will receive this increase in welfare.
- The cost of the increase in ϖ is the decline in $F(\theta^*)$, the probability with which the citizens will receive this amount, times ϖ. This decline in probability is equal to $f(\theta^*)/g^*(\theta^*)$.

Agency with Asymmetric Information (continued)

- Rearranging the previous condition, we have

$$\frac{\varpi}{g^*(\theta^*)} = \frac{F(\theta^*)}{f(\theta^*)} \tag{8}$$

or alternatively

$$\frac{d\varpi/d\theta^*}{\varpi} = -\frac{f(\theta^*)}{F(\theta^*)}$$

- Of course, F/f or f/F are familiar and captured the hazard rate or the relative likelihood that $\theta = \theta^*$ conditional on $\theta \leq \theta^*$.
- Under the assumption that the hazard rate is monotonically decreasing, which is another standard restriction, (8), defines a unique θ^*.
- You may note that this assumption also corresponds to the assumption that $F(\theta)$ is log concave. Most familiar distribution functions such as uniform, normal etc. satisfy this assumption.

Agency with Asymmetric Information (continued)

- The monotone likelihood ratio assumption implies that the right hand side of (8) is decreasing in θ^*, while the fact that $g^*(\theta^*)$ is decreasing makes the left-hand side increasing, so θ^* is uniquely defined.
- Now, consider a change in the distribution of the costs of public goods from $F(\theta)$ to $\tilde{F}(\theta)$ such that $F(\theta^*) < \tilde{F}(\theta^*)$ and $f(\theta^*) = \tilde{f}(\theta^*)$, so that the probability that $\theta \leq \theta^*$ has increased, but the density of θ around θ^* has not.
- In this case, the first-condition above implies that ϖ should increase, since there is a higher likelihood of any given level of ϖ to be delivered to the citizens.

Agency with Asymmetric Information (continued)

- What is the effect of asymmetric information on the provision of public goods (or voters ex post welfare). Recall that with symmetric information we had

$$g^F(\theta) = g^*(\theta)$$

- In contrast, with incomplete information, we have

$$g^I(\theta) = \begin{cases} 0 & \text{for } \theta > \theta^* \\ g^*(\theta) & \text{for } \theta \leq \theta^* \end{cases}$$

- Notice that, when $\theta \leq \theta^*$, the provision of the public good is not distorted because of the form of the voting rule (and quasi-linearity of preferences).
- Therefore, $g^I(\theta) \leq g^F(\theta)$.

Agency with Asymmetric Information (continued)

- What about rents?

$$r(\theta) = \begin{cases} y & \text{for } \theta > \theta^* \\ r^* + (\tau^*(\theta^*) - \tau^*(\theta))y + H(g^*(\theta)) - H(g^*(\theta^*)) & \text{for } \theta \leq \theta^* \end{cases}$$

- Since both $y > r^*$ and $(\tau^*(\theta^*) - \tau^*(\theta))y + H(g^*(\theta)) - H(g^*(\theta^*)) \geq 0$, politicians always receive greater rents with asymmetric information.
- This is intuitive: asymmetric information increases the control power of politicians (because voters cannot monitor them as well).

Electoral Controls with Career Concerns

- A major shortcoming of the previous models is that voters use retrospective voting rules, rather than pursue policies that are in their future interests.
- One way of overcoming this problem is to look at a different type of agency models, inspired by Holmstrom's career concerns model.
- The main idea is that there is a feature of the politician, like ability, that voters care about, which also affects outcomes.
- Then, forward-looking voters look at past performance to estimate the ability—and thus the electability—of the politician.

Career Concerns (continued)

- Let us illustrate the main issues using a two-period model here.
- To simplify the model, let us assume that taxes are fixed at $\bar{\tau}$, so the only problem is to make sure that the tax revenue is spent on public goods.
- The welfare of the voters is again

$$U_t = \bar{y}(1 - \bar{\tau}) + g_t.$$

- Now the technology for public good provision is different, and takes the form

$$g_t = \eta(\bar{\tau}\bar{y} - r_t),$$

  where η is the "ability" of the politician, which is fixed in both periods.
- Let us assume that it is drawn uniformly from the interval

$$\left[1 - \frac{1}{2\xi}, 1 + \frac{1}{2\xi}\right].$$

Career Concerns (continued)

- The important simplifying assumption of the Holmstrom model, which we adopt here, is that there is symmetric information, so the politician is also uncertain about η with the same prior.
- The utility of the politician is a simple extension of what we had before:

$$v_I = r_1 + p_I \beta (R + r_2),$$

  with $0 < \beta < 1$ again as the discount factor, and now R is interpreted as non-pecuniary grants from being in power.
- Moreover, let us assume that there is a maximum on the rents that the politician can extract, $\bar{r}$.
- The exact timing of events is as follows:
  - Nature determines η.
  - The politician chooses r_1.
  - Observing g_1 (but not r_1), voters decide whether to keep the politician. If they elect a new politician, he is drawn randomly from the same distribution.
  - The politician and power chooses g_2.

Career Concerns (continued)

- Given this structure, the equilibrium is straightforward to determine.
- In the second period, there is no control over the politician, so he will set

$$r_2 = \bar{r},$$

and public goods will be

$$g_2 = \eta(\bar{\tau} y - \bar{r})$$

- If they appoint a new politician, he will have $\mathbb{E}(\eta) = 1$, so the expected utility of appointing a new politician for the voters is

$$U_2^N = \bar{y}(1 - \bar{\tau}) + (\bar{\tau} y - \bar{r})$$

- What about the utility of keeping the incumbent? This would be

$$U_2^I = \bar{y}(1 - \bar{\tau}) + \tilde{\eta}(\bar{\tau} y - \bar{r})$$

where $\tilde{\eta}$ is their posterior about his ability.

Career Concerns (continued)

- Now suppose that voters know that the politician will choose $\tilde{r}_1$ amounts of rents for himself.
- Then they can estimate
$$\tilde{\eta} = \frac{g_1}{\bar{\tau} y - \tilde{r}_1}$$
and their optimal reelection decision is
$$\tilde{p}_I = \begin{cases} 1 & \text{iff } \tilde{\eta} \geq \mathbb{E}(\eta) = 1 \\ 0 & \text{otherwise.} \end{cases}.$$
- The problem is that $\tilde{r}_1$ is an equilibrium choice by the politician.
- He will try to make this choice in order to ensure that he remains in power if this is beneficial for him.
- This is why this class of models are sometimes called "signal jamming" models.

Career Concerns (continued)

- To make more progress, let us first look at the probability that he keeps power.
- This is

$$\begin{aligned} p_I &= \text{Prob}\left[\tilde{p}_I = 1\right] = \text{Prob}\left[\tilde{\eta} \geq 1\right] \\ &= \text{Prob}\left[\frac{g_1}{\bar{\tau}\bar{y} - \tilde{r}_1} \geq 1\right] = \text{Prob}\left[\frac{\eta(\bar{\tau}\bar{y} - r_1)}{\bar{\tau}\bar{y} - \tilde{r}_1} \geq 1\right] \\ &= \frac{1}{2} + \xi\left[1 - \frac{\eta(\bar{\tau}\bar{y} - r_1)}{\bar{\tau}\bar{y} - \tilde{r}_1}\right] \end{aligned}$$

  where the last equality exploits the uniform assumption.
- Now the incumbent will choose r_1 to maximize $v_I = r_1 + p_I \beta(R + r_2)$, which we can write as:

$$\max_{r_1} v_I = r_1 + \left[\frac{1}{2} + \xi\left(1 - \frac{\eta(\bar{\tau}\bar{y} - r_1)}{\bar{\tau}\bar{y} - \tilde{r}_1}\right)\right]\beta(R + \bar{r})$$

Career Concerns (continued)

- The first-order condition of this maximization problems gives

$$1 - \frac{\xi(\bar{\tau}\bar{y} - \tilde{r}_1)}{(\bar{\tau}\bar{y} - r_1)^2}\beta(R + \bar{r}) = 0 \tag{9}$$

- This defines a best-response $r_1(\tilde{r}_1)$ by the incumbent. When voters expect them to play $\tilde{r}_1$, he would play $r_1(\tilde{r}_1)$.
- Clearly, the equilibrium has to be a fixed point, $r_1(\tilde{r}_1) = \tilde{r}_1$.
- Substituting this into (9), we obtain

$$r_1 = \bar{\tau}y - \xi\beta(R + \bar{r})$$

  and the politician keeps power with probability $p_I = \frac{1}{2}$, since in equilibrium nobody's fooled, and with probability 1/2 the politician is worse than average.

Career Concerns (continued)

- This is therefore a more satisfactory model of deriving results in which elections appear as a method of controlling politicians.
- In fact, we can alternatively express this equilibrium as remarking that voters will reelect the incumbent only if

$$g_1 = \xi\beta(R + \tilde{r})\eta \geq \xi\beta(R + \bar{r})$$

  or their utility exceeds a certain threshold as in the retrospective voting models:

$$\bar{y}(1 - \bar{\tau}) + \xi\beta(R + \tilde{r})\eta \geq \varpi \equiv \bar{y}(1 - \bar{\tau}) + \xi\beta(R + \bar{r}).$$

- Nevertheless, it is important to emphasize that such rules are not retrospective or pre-determined punishment rules; instead they are derived from the forward-looking optimum behavior of the voters.

Dynamic Political Agency

- Let us now relax the assumption that citizens have to vote retrospectively and that they have to use stationary strategies.
- Therefore, we will look for subgame perfect equilibria in a political agency model.
- In particular, since the focuses on how well electoral controls work, let us look for the "best subgame perfect equilibrium" from the viewpoint of the citizens.
- Since the best subgame perfect equilibrium will be similar to a "mechanism," let us refer to this as *the best sustainable mechanism.*
- Also, let us enrich the environment so that there is labor supply and capital accumulation as in the standard *neoclassical model*, so that distortions created by political economy can be seen more transparently.

Dynamic Political Agency (continued)

- The model is in infinite horizon and discrete time, and it is populated by a continuum of measure 1 of identical individuals (citizens). Individual preferences at time $t = 0$ are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

where c denotes consumption and l is labor supply.

- Denote the set of citizens by I and use the subscript i to denote citizens.

Assumption: $U(c, l)$ is twice continuously differentiable with partial derivatives denoted by U_C and U_L, strictly increasing in c, strictly decreasing in l and jointly concave in c and l. We adopt the normalization $U(0, 0) = 0$. Moreover, $l \in [0, \bar{L}]$.

Dynamic Political Agency (continued)

- The production side of the economy is described by the aggregate production function

$$Y_t = F(K_t, L_t), \tag{10}$$

which is defined inclusive of undepreciated capital (i.e., $F(K_t, L_t) \equiv \tilde{F}(K_t, L_t) + (1-\theta) K_t$ for some other production function $\tilde{F}(K, L)$ and for some depreciation rate $\theta \in (0, 1)$).
Assumption: F is strictly increasing and continuously differentiable in K and L with partial derivatives denoted by F_K and F_L, exhibits constant returns to scale, and satisfies $\lim_{L \to 0} F_L(K, L) = \infty$ for all $K \geq 0$ and $\lim_{K \to \infty} F_K(K, L) < 1$ for all $L \in [0, \bar{L}]$.
- The condition that $\lim_{K \to \infty} F_K(K, L) < 1$ together with $L \in [0, \bar{L}]$ implies that there is a maximum steady-state level of output is uniquely defined by $\bar{Y} = F(\bar{Y}, \bar{L}) \in (0, \infty)$.
- The condition that $\lim_{L \to 0} F_L(K, L) = \infty$ implies that in the absence of distortions there will be positive production.

Dynamic Political Agency (continued)

- As in the baseline political agency model, the allocation of resources is delegated to a politician (ruler).
- Suppose that there is a large number of potential (and identical) politicians, denoted by the set $\mathcal{I}$.
- Each politician's utility at time t is given by

$$\sum_{s=0}^{\infty} \delta^s v\left(x_{t+s}\right),$$

  where x denotes the politician's consumption (rents) and $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ is his instantaneous utility function.
- Notice also that the politician's discount factor, δ, is potentially different from that of the citizens, β.
- To simplify the analysis, we assume that potential politicians are distinct from the citizens and never engage in production and that once they are replaced they do not have access to capital markets.

Dynamic Political Agency (continued)

- **Assumption:** v is twice continuously differentiable, concave, and satisfies $v'(x) > 0$ for all $x \in \mathbb{R}_+$ and $v(0) = 0$. Moreover $\delta \in (0, 1)$.
- The politician in power decides the allocation of resources (or equivalently decides a general set of taxes and transfers).
- The only restriction on the allocation of resources, in addition to $c_t \geq 0$ and $l_t \in [0, \bar{L}]$, comes from the *participation constraint* of the citizens, which requires that $U(c_t, l_t) \geq 0$ for each t. We denote the three constraints $c_t \geq 0$, $l_t \in [0, \bar{L}]$ and $U(c_t, l_t) \geq 0$ by

$$(c_t, l_t) \in \Lambda \text{ for all } t. \tag{11}$$

- Since $U(c, l)$ is concave and continuous, Λ is closed and convex (and also nonempty). We use IntΛ to denote the interior of the set Λ, so that $(c_t, l_t) \in$IntΛ implies that $c_t > 0$, $l_t \in (0, \bar{L})$ and $U(c_t, l_t) > 0$.

Dynamic Political Agency (continued)

- The political game is as follows. At each time t, the economy starts with a politician $\iota_t \in \mathcal{I}$ in power and a stock of capital inherited from the previous period, K_t. Then:
  - Citizens make labor supply decisions, denoted by $[l_{i,t}]_{i \in I}$, where $l_{i,t} \geq 0$. Output $F(K_t, L_t)$ is produced, where $L_t = \int_{i \in I} l_{i,t} di$.
  - The politician chooses the amount of rents $x_t \in \mathbb{R}_+$, a consumption function $\mathbf{c}_t : \mathbb{R}_+ \to \mathbb{R}_+$, which assigns a level of consumption for each level of (current) labor supply, and next period's capital stock $K_{t+1} \in \mathbb{R}_+$, subject to the constraint

$$K_{t+1} \leq F(K_t, L_t) - C_t - x_t,$$

    where $C_t = \int_{i \in I} \mathbf{c}_t(l_{i,t}) \, di$ is aggregate consumption. We denote a triple $(x_t, \mathbf{c}_t, K_{t+1})$ that is feasible for the politician by $(x_t, \mathbf{c}_t, K_{t+1}) \in \Phi_t$.
  - Elections are held and citizens jointly decide whether to keep the politician or replace him with a new one, $\rho_t \in \{0, 1\}$, where $\rho_t = 1$ denotes replacement.

Dynamic Political Agency (continued)

- The important feature here is that even though individuals make their economic decisions independently, they make their political decisions—elections to replace the politician—jointly.
- This is natural since there is no conflict of interest among the citizens over the replacement decision. Joint political decisions might be achieved by a variety of procedures, including various voting schemes. Here let us simply assume that the decision $\rho_t \in \{0, 1\}$ is taken by a randomly chosen citizen.
- Let us also assume that at each date there is a public random variable z_t and all agents can condition their strategies on the history of this variable (this is only useful for convexitication).
- Let

  $$h^t \equiv \left(K_0, \iota_0, z_0, [l_{i,0}]_{i \in I}, x_0, \mathbf{c}_0, \rho_0, K_1, \ldots K_t, \iota_t, z_t, [l_{i,t}]_{i \in I}, x_t, \mathbf{c}_t, \rho_t, K_{t+1}\right.$$

  denote the history of the game up to date t, and H^t be the set of all such histories.
- To simplify notation we suppress the conditioning on the history of z^t.

Dynamic Political Agency (continued)

- A *subgame perfect equilibrium* (SPE) is given by labor supply decisions $\left[l_{i,t}^{*}\right]_{i \in I}$ at time t given history h^{t-1}, policy decisions $x_t^*, \mathbf{c}_t^*, K_{t+1}^*$ by the politician in power given h^{t-1} and $[l_{i,t}]_{i \in I}$, and electoral decisions by the citizens, ρ_t^* at time t, given history h^{t-1} and $[l_{i,t}]_{i \in I}$, $x_t^*, \mathbf{c}_t^*, K_{t+1}^*$ that are best responses to each other for all histories. In addition, we will show below that the SPE we focus on are "renegotiation-proof".
- Let us say that a SPE is *renegotiation-proof* if after any history h^t there does not exist another SPE that can make all active players weakly better off (and some strictly better off). In the present context, this implies that there should not exist an alternative SPE that can make the citizens and the politician in power better off than in the candidate SPE.

Dynamic Political Agency (continued)

- We focus on *best SPE*, defined as a SPE that maximizes the utility of the citizens.
- Consider the following constrained optimization problem:

$$\textbf{MAX}: \quad \max_{\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{12}$$

  subject to the participation constraint (11), the resource constraint,

$$C_t + K_{t+1} + x_t \leq F(K_t, L_t) \text{ for all } t, \tag{13}$$

  the sustainability constraint for the politician in power,

$$w_t \equiv \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v(F(K_t, L_t)) \text{ for all } t, \tag{14}$$

  and given the initial capital stock $K_0 > 0$.
- Capital letters because consumption and labor supply levels refer to both individuals and aggregates.

Dynamic Political Agency (continued)

- The sustainability constraint, (14), requires the equilibrium utility of the politician to be such that he does not wish to choose the maximum level of rents this period, $x_t = F(K_t, L_t)$, which would give him utility $v(F(K_t, L_t))$.
- Let us refer to a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that is a solution to this problem as a *best sustainable mechanism* (since it implicitly defines a resource allocation mechanism).
- The constraint, (14), is sufficient to ensure that the politician does not wish to deviate from the mechanism.

Dynamic Political Agency (continued)

- **Proposition:** The allocation of resources in the best SPE (best sustainable mechanism) is identical to the solution of the maximization problem in *(MAX)* and involves no replacement of the initial politician along the equilibrium path. Moreover, this allocation can be supported as a renegotiation-proof SPE.
- This proposition enables us to focus on the constrained maximization problem given in (MAX).
- Moreover, it implies that in the best SPE, the initial politician will be kept in power forever (and that this best SPE is renegotiation-proof): the initial politician is kept in power forever because all politicians are identical and more effective incentives can be provided to a politician when he has a longer planning horizon (i.e., when he expects to remain in power for longer). Naturally, he is only kept in power along the equilibrium path—if he deviates from the implicitly-agreed mechanism, he will be replaced.

Dynamic Political Agency (continued)

- For future reference, let us define an *undistorted* allocation as a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that maximizes (12) without the sustainability constraint (14) (for a given sequence of $\{x_t\}_{t=0}^{\infty}$).
- An undistorted allocation where $(C_t, L_t) \in \text{Int}\Lambda$ satisfies

$$F_L(K_t, L_t)\, U_C(C_t, L_t) = -U_L(C_t, L_t), \tag{15}$$

$$U_C(C_t, L_t) = \beta F_K(K_{t+1}, L_{t+1})\, U_C(C_{t+1}, L_{t+1}). \tag{16}$$

- We say that an allocation $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ features *downward labor distortions* at time t if the left-hand side of (15) is strictly greater than the right-hand side.
- Similarly, there are *downward intertemporal distortions* when the left-hand side of (16) is strictly less than the right-hand side.
- Downward distortions imply that there is less labor supply and less capital accumulation than in an undistorted allocation.

Best Sustainable Mechanism without Capital

- Let us start with the economy without capital, so that

$$Y_t = L_t.$$

- An allocation can now be represented by $\{C_t, L_t, x_t\}$ (thus dropping K_t). An undistorted allocation with $(C_t, L_t) \in \text{Int}\Lambda$ now satisfies $U_C(C_t, L_t) = -U_L(C_t, L_t)$.
- The next assumption ensures that when the maximum amount of utility is given to the politician in every period, this is sufficient to satisfy the sustainability constraint (14):
  Assumption: Let $(\tilde{C}, \tilde{L}) \in \arg\max_{(C,L)\in\Lambda} \{L - C\}$. Then, $v(\tilde{L} - \tilde{C}) / (1 - \delta) > v(\tilde{L})$.

Best Sustainable Mechanism (continued)

Theorem

Suppose that $Y_t = L_t$*, that the above assumptions hold and that* $U_C(0,0) > U_L(0,0)$*. Then in the best SPE (best sustainable mechanism), we have:*

1. there are downward labor distortions at $t = 0$*.*

2. when $\beta \leq \delta$*, the values promised to the politician* $\{w_t\}_{t=0}^{\infty}$ *form a nondecreasing sequence and converge to some* w^**. Moreover,* $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ *converges to some* (C^*, L^*, x^*)*, which satisfies the no-distortion condition* $U_C(C^*, L^*) = -U_L(C^*, L^*)$*.*

3. when $\beta > \delta$*, then there are downward labor distortions even asymptotically.*

The allocation described above can be supported as a renegotiation-proof SPE.

Intuition

- Part 1 of the theorem illustrates the additional distortion arising from the sustainability constraints.
- As output increases, the sustainability constraint, (14), requires more rents to be given to the politician in power and this increases the effective cost of production for the citizens.
- The best SPE creates distortions so as to reduce the level of output and thus the rents that have to be paid to the politician.
- Starting from an undistorted allocation, this is always beneficial. Loosely speaking, a marginal distortion, reducing labor supply and output by a small amount, creates a "second-order" loss for the citizens, but a "first-order" reduction in the amount of rents that have to be paid to the politician and thus a "first-order" increase in their consumption and utility.

Intuition (continued)

- Part 2 states that as long as $\beta \leq \delta$, the economy asymptotically converges to an equilibrium (C^*, L^*, x^*) where there are no aggregate distortions; even though there will be rents provided to the politician, these will be financed without introducing distortions.
- Therefore,there will be "efficient" provision of rents to politicians, with the necessary tax revenues raised without distortions (e.g., with lump-sum taxes in a decentralized allocation).
- This part of the theorem also shows that the (promised) rewards to the politician, given by the sequence $\{w_t\}_{t=0}^{\infty}$, are nondecreasing. Intuitively, current incentives to the politician are provided by both consumption in the current period, x_t, and by consumption in the future represented by the promised value, w_{t+1}.
- Future consumption by the politician not only relaxes the sustainability constraint in the future but does so in all prior periods as well. Thus, all else equal, optimal incentives for the politician should be backloaded.

Intuition (continued)

- Part 3 of the theorem states that if the politicians are less patient than the citizens, distortions will never disappear.
- Since in many realistic political economy models politicians are—or act—more short-sighted than the citizens, this part of the theorem implies that in a number of important cases, political economy considerations will lead to additional distortions that will not disappear even asymptotically.
- Finally, Theorem 1 also shows that the best SPE can be supported as a renegotiation-proof equilibrium.

Sketch Proof

- Let us represent the maximization problem in (MAX) recursively:

$$V(w) = \max_{(C,L)\in\Lambda,x,w^+} \left\{ U(C,L) + \beta V(w^+) \right\} \quad (17)$$

subject to

$$C + x \leq L, \quad (18)$$

$$w = v(x) + \delta w^+, \quad (19)$$

$$v(x) + \delta w^+ \geq v(L). \quad (20)$$

- Here $V(w)$ is the value of the citizens when they have promised value w to the politician and w^+ denotes next period's promised value.
- Constraint (18) imposes the resource constraint (13).
- Constraint (19) imposes promise keeping, incorporating the fact that the politician will not be replaced. It requires that the promised value w be equal to the sum of $v(x)$ and the continuation utility, δw^+.
- Finally, constraint (20) is the recursive version of (14).

Sketch Proof (continued)

- Let γ and $\psi \geq 0$ be the Lagrange multipliers on the constraints (19) and (20) respectively.
- It can be shown that $V(w)$ is concave and differentiable.
- Furthermore, for the intuitive argument here, suppose that $(C, L) \in \text{Int}\Lambda$. The first-order condition with respect to w^+ and the envelope theorem then imply

$$\frac{\beta}{\delta} V'(w^+) = -\gamma - \psi = V'(w) - \psi. \tag{21}$$

- Combining the first-order conditions for C and L gives

$$U_C(C, L) + U_L(C, L) = \psi v'(L). \tag{22}$$

Sketch Proof (continued)

- Equation (22) makes it clear that aggregate distortions are related to the Lagrange multiplier on the sustainability constraint, ψ.
- Moreover, we must have $\psi > 0$ at $t = 0$, otherwise the politician would receive $w_0 = 0$ initially, which together with (20) would imply $C_t = L_t = 0$ for all t.
- However, $C_t = L_t = 0$ for all t cannot be a solution when $\psi = 0$ at $t = 0$. Equation (22) then yields $U_C(C, L) + U_L(C, L) > 0$ at $t = 0$.

Sketch Proof (continued)

- To obtain the intuition for the second part of Theorem 1, consider the case where $\beta = \delta$.
- Then, equation (21) implies

$$V'(w^+) = V'(w) - \psi \leq V'(w). \tag{23}$$

- Concavity of the value function $V(\cdot)$ then implies that $w^+ \geq w$, with $w^+ > w$ if $\psi > 0$, and $w^+ = w$ if $\psi = 0$.
- Therefore, the values promised to the politician form a nondecreasing sequence and converge to some w^* and (23) implies that ψ must converge to 0.
- This also implies that $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ converges to some (C^*, L^*, x^*), which satisfies (20) as stated in part 2 of Theorem 1.

Sketch Proof (continued)

- This argument breaks down in part 3 of the theorem when $\delta < \beta$ because the politician does not value future rewards sufficiently and the sequence $\{w_t\}_{t=0}^{\infty}$ is not necessarily nondecreasing.
- In fact, (21) implies that if $\{w_t\}_{t=0}^{\infty}$ converges to some $\hat{w}$, then $\beta V'(\hat{w})/\delta = V'(\hat{w}) - \psi$.
- Since V' is negative, ψ must be strictly positive in this case and there will necessarily be asymptotic distortions.
- Therefore, in this case distortions from political economy remain forever.

Best Sustainable Mechanism with Capital

- Let us next extend Theorem 1 to an environment with capital.
- Let us define $\bar{C}$ and $\bar{K}$ such that

$$\bar{C} = \min\{C : (C, \bar{L}) \in \Lambda\} \text{ and } \bar{K} = \arg\max_{K \geq 0} \{F(K, \bar{L}) - K - \bar{C}\}. \tag{24}$$

Clearly, $\bar{C}$ is uniquely defined (since $C \geq 0$ and Λ is closed). In view of this and Assumption 2, $\bar{K}$ is also uniquely defined.

Assumption:
(1) $\delta v(F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}) / (1 - \delta) > v(F(\bar{K}, \bar{L}))$; and (2) $\bar{C} + \bar{K} \leq F(0, \bar{L})$.

Main Result

Theorem

Suppose that the above assumptions hold. Then in the best SPE:
1. there are downward labor distortions at some $t < \infty$ *and downward intertemporal distortions at* $t - 1$ *(provided that* $t \geq 1$*);*
2. when $\beta \leq \delta$*, the best sustainable mechanism* $\{C_t, K_{t+1}, L_t, x_t\}_{t=0}^{\infty}$ *converges to some* (C^*, K^*, L^*, x^*)*. At this allocation, the labor and intertemporal distortions disappear asymptotically, i.e., (15) and (16) hold as* $t \longrightarrow \infty$*;*
3. when $\beta > \delta$*, there are downward labor and intertemporal distortions, even asymptotically.*
The allocation described above can be supported as a renegotiation-proof SPE.

Interpretation

- This theorem generalizes the results of Theorem 1 to an environment that is identical to the standard neoclassical growth model.
- When $\beta > \delta$, the best SPE not only generates labor distortions but also intertemporal distortions.
- These can be thought of as "aggregate capital taxes," since they create a wedge between the marginal product of capital and the ratio of marginal utilities of consumption.
- Therefore, this model generates a political economy rationale for long-run capital taxation.

Stationary Equilibria

- Let us not return to stationary equilibria as the typical applications of the Barro-Ferejohn approach assume.
- Let us focus on best *stationary* SPE and ignore capital.
- With stationary strategies and no capital, x_t has to be constant (conditional on the politician remaining in power).
  Proposition: Consider the environment without capital in Theorem 1 and suppose that the above assumptions hold and that $U_C(0,0) > U_L(0,0)$. Then, in the best stationary SPE distortions never disappear.

Stationary Equilibria (continued)

- Why? Because with a stationary equilibrium path, $x_t = x$ and $L_t = L$ so that
$$\frac{v(x)}{1-\delta} \geq v(L) \tag{25}$$
replaces the sustainability constraint (14).
- Constraint (25) must bind in all periods with $\psi > 0$, since otherwise the solution to the stationary equivalent of (MAX) would involve $x = 0$ and no distortions.
- The assumption that $U_C(0,0) > U_L(0,0)$ then implies that in this case $L > 0$, thus $x = 0$ would violate (25).
- Condition (22), which still applies in this case, then shows that there is a positive distortion on labor in all periods.

Stationary Equilibria (continued)

- This proposition illustrates the role of nonstationary SPE in the results above and also highlights why the restriction to stationary strategies in the standard models might be restrictive.
- Stationary equilibria do not allow the optimal provision of dynamic incentives to politicians and imply that political economy distortions never disappear, even when $\beta \leq \delta$.