Political Economy of Institutions and Development: 14.773 Problem Set 5

This problem set is not due. It is for practice only.

Question 1

Suppose that the economy consists of two groups, the elite and the producers. Suppose that both groups are of equal size (each consisting of a continuum of agents). Both groups have instantaneous utility $u(c) = \log c$ and discount factor β. Producers have access to the production technology $f(k) = Ak^{\alpha}$, where k is capital. The elite impose a linear tax rate of τ_t on production at time t and consume the proceeds. The capital stock for time t must be chosen at time t after the tax rate τ_t has been announced. There is full depreciation of capital.

1. Given a tax sequence, set up the dynamic optimization problem of entrepreneurs and show that the evolution of the capital stock is given by
$$k_{t+1} = \alpha\beta(1-\tau_t)Ak_t^{\alpha}. \quad (1)$$
Explain why the capital stock for time $t+1$ does not depend on the current tax rate but only on the past tax rate. [Hint: to derive this, set up the maximization problem of the entrepreneur is a dynamic program and conjecture a decision rule of the form $k_{i,t+1} = \kappa y_{i,t}$ for entrepreneur i, where $y_{i,t}$ is his output at time t].

2. To determine the Markov Perfect Equilibrium tax rates, write the value to a representative elite agent at time $t+1$ as a function of the tax rate $\tau = \tau_{t+1}$, taking into account that the capital stock of entrepreneurs at date $t+1$, $k = k_{t+1}$ is given from (1). Show that this value function takes the form
$$W(k) = \max_{\tau\in[0,1]} \left\{\log\left[\tau Ak^{\alpha}\right] + \beta W\left(\alpha\beta(1-\tau)Ak^{\alpha}\right)\right\}. \quad (2)$$

Use standard dynamic programming arguments show that W is strictly concave and differentiable for $k > 0$ (and denote the derivative by W'). Show that the Euler equation for the elite is

$$\frac{1}{\tau} = \beta^2 \alpha A k^{\alpha} W'\left(k'\right) = \beta \frac{k' W'\left(k'\right)}{1-\tau}.$$

3. Now conjecturing that $W(k) = \eta + \gamma \log k$ and using the Envelope condition, show that $\gamma = \alpha/(1-\alpha\beta)$ and derive the law of motion of the capital stock of each entrepreneur (and the aggregate capital stock). Explain the role of logarithmic preferences in this result.

Question 2

Consider an economy populated by λ rich agents who initially hold power, and $1-\lambda$ poor agents who are excluded from power, with $\lambda < 1/2$. All agents are infinitely lived and discount the future at the rate $\beta \in (0,1)$. Each rich agent has income θ/λ while each poor agent has income $(1-\theta)/(1-\lambda)$ where $\theta > \lambda$. The political system determines a linear tax rate, τ, the proceeds of which are redistributed lump-sum. Each agent can hide their money in an alternative non-taxable production technology, and in the process they lose a fraction ϕ of their income. There are no other costs of taxation. The poor can undertake a revolution, and if they do so, in all future periods, they obtain a fraction $\mu(t)$ of the total income of the society (i.e., an income of $\mu(t)/(1-\lambda)$ per poor agent). The poor cannot revolt against democracy. The rich lose everything and receive zero payoff after a revolution. At the beginning of every period, the rich can also decide to extend the franchise to the poor, and this is irreversible. If the franchise is extended, the poor decide the tax rate in all future periods.

1. Define MPE in this game.

2. First suppose that $\mu(t) = \mu^l$ at all times. Also assume that $0 < \mu^l < 1-\theta$. Show that in the MPE, there will be no taxation when the rich are in power, and the tax rate will be $\tau = \phi$ when the poor are in power. Show that in the MPE, there is no extension of the franchise and no taxation.

3. Suppose that $\mu^l \in (1-\theta, (1-\phi)(1-\theta) + \phi(1-\lambda))$. Characterize the MPE in this case. Why is the restriction $\mu^l < (1-\phi)(1-\theta) + \phi(1-\lambda)$ necessary?

4. Now consider the SPE of this game when $\mu^l > 1 - \theta$. Construct an equilibrium where there is extension of the franchise along the equilibrium path. [Hint: first, to simplify, take $\beta \to 1$, and then consider a strategy profile where the rich are always expected to set $\tau = 0$ in the future; show that in this case the poor would undertake a revolution; also explain why the continuation strategy of $\tau = 0$ by the rich in all future periods could be part of a SPE]. Why is there extension of the franchise now? Can you construct a similar non-Markovian equilibrium when $\mu^l < 1 - \theta$?

5. Explain why the MPE led to different predictions than the non-Markovian equilibria. Which one is more satisfactory?

6. Now suppose that $\mu(t) = \mu^l$ with probability $1 - q$, and $\mu(t) = \mu^h$ with probability q, where $\mu^h > 1 - \theta > \mu^l$. Construct a MPE where the rich extend the franchise, and from there on, a poor agent sets that tax rate. Determine the parameter values that are necessary for such an equilibrium to exist. Explain why extension of the franchise is useful for rich agents?

7. Now consider non-Markovian equilibria again. Suppose that the unique MPE has franchise extension. Can you construct a SPE equilibrium, as $\beta \to 1$, where there is no franchise extension?

8. Contrast the role of restricting strategies to be Markovian in the two cases above [Hint: why is this restriction ruling out franchise extension in the first case, while ensuring that franchise extension is the unique equilibrium in the second?].

Question 3

Consider a society consisting of $|N|$ groups, and denote the set of groups by N. Each group is identified by its political (voting) power $\gamma_i \in \mathbb{R}_+$. Throughout suppose that the groups have solved all of their internal collective action problems and act as single agent in the game.

The society has a resource normalized to 1, which will be divided between these groups. Let an allocation of resources between the groups be $\{x_i\}_{i \in N}$ such that $x_i \geq 0$ for all i and $\sum_{i \in 1} x_i \leq 1$. Each group i has strictly increasing preferences over their share of the resource, x_i. Consider the following game:

- The group with the highest γ_i, group N, makes an allocation offer $\{x_i\}_{i \in N}$.

- All groups vote over this allocation. Each group's vote is equivalent to its power γ_i. If the allocation is accepted (i.e., $\sum_{i\in \text{Yes}} \gamma_i > \sum_{i\in \text{No}} \gamma_i$, where Yes denotes the set of groups that vote yes and No denotes the set of groups that vote no), it is implemented. Otherwise, we move to the next stage, and the next group with the highest power makes an offer.

- The game continues until one of the offers is accepted or the last group makes an offer. If the last offer is also rejected, then all groups receive the allocation $\left\{x_i^d\right\}_{i\in N}$ such that $x_i^d \geq 0$ for all i and $\sum_{i\in 1} x_i^d \leq 1$. There is no discounting.

1. Explain why this game has a subgame perfect equilibrium. Is it necessarily unique? Can you provide a condition that guarantees uniqueness?

2. Explain why if $\sum_{i\in 1} x_i^d = 1$, in any subgame perfect equilibrium, the equilibrium outcome must satisfy $x_i = x_i^d$ for all $i \in N$.

3. Next suppose that $x_i^d = 0$ for all $i \in N$. Suppose also that the society consists of five groups with $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = 4$, $\gamma_4 = 8$ and $\gamma_5 = 9.5$. Characterize the subgame perfect equilibria of this game.

4. Now suppose that groups cannot make offers of the form $\{x_i\}_{i\in N}$. Instead, they can make coalition offers. For example, group 5 can make an offer of $\{1, 2, 5\}$, which would mean that these three groups form a ruling coalition. A ruling coalition divides the resource among its members according to their power. That is, if X is in the ruling coalition, each $i \in X$ receives $x_i = \gamma_i / \sum_{j\in X} \gamma_j$. Find the subgame perfect equilibrium of this game with the five groups as specified in part 3.

5. Discuss what would happen in part 4 if once a coalition is formed, there is one more round of voting within the coalition to form yet another ruling subcoalition. For example, if the original coalition was $\{1, 2, 5\}$, group 5 can now offer $\{5\}$ as the ruling coalition, and win against groups 1 and 2 (since it has 5 votes versus their 3 combined). Can you find the unique ruling coalition in this case?

6. Let us now return to the original game with offers and modify the game form so that after the least powerful group makes an offer and it is rejected, the cycle of offers starts again. Payoffs are only realized once

agreement is reached and at that point the game ends (it continues indefinitely if no offer is accepted). Does there exist a subgame perfect equilibrium. If yes, give an example. If no, explain why not. Is there any relationship between this game and the non-existence of the core in cooperative games of resource division?

7. Now consider an infinite-horizon version of the game discussed in 5 above, where at each stage the current coalition can votes to include further members or exclude existing members. At the end of each date, payoffs are realized as specified in 4 above (i.e., each $i \in X$ receives $x_i = \gamma_i / \sum_{j \in X} \gamma_j$). Relate this game to Kevin Roberts' model of "Voting in Clubs". In what ways is it more general? In what ways is it more specific? What types of problems do you for see if you were to try to characterize the Markov perfect equilibria of this game. [*Do not attempt to characterize such equilibria.* Simply state what the difficulties would be if you tried to characterize them and if you have ideas about how to overcome these difficulties, feel free to suggest them].

8. Discuss whether games of this form may be useful in understanding coalition formation in real-world political environments. For example, consider the following concrete situation. There are three groups; unionized workers, bankers and monopolists, and they are trying to decide over labor market reforms (disliked by unionized workers), financial reforms (disliked by bankers) and product market reforms (disliked by monopolists). Suppose that each reform is liked by groups that do not dislike it and if two groups agree, they can become the ruling coalition and decide on all the reforms. Could you use a model along the lines of that described above to decide what type of coalition will form and which reforms will pass? How would you enrich or simplify the model here if you wanted to make more progress on this specific question?

Question 4

Consider a country consisting of two ethnic groups, A and B. All agents are infinitely lived (in discrete time) and maximize the net present discounted value of their income with discount factor $\beta \in (0, 1)$. Suppose that both groups are of equal size, and have exogenous income levels y_j, $j \in \{A, B\}$ in each period. At the beginning of the period t, there are two possible political regimes S_{t-1} inherited from yesterday. A-dominance

($S_{t-1} = A$) and B-dominance ($S_{t-1} = B$). A secession shock $x_t = \{0, A, B\}$ takes place with probabilities $\{1 - q_A - q_B, q_A, q_B\}$ where $q_A, q_B < 1/2$, where x_t denotes the identity of the group which will have the opportunity to secede at the end of the period. Whoever has political power as determined by S_{t-1} chooses S_t which determines the group which can set policies today (and institutions S_{t+1} tomorrow), where policies consist of a policy vector $\left(\tau_t^A, \tau_t^B\right)$, where τ_t^j is a lump-sum tax imposed on group j, satisfying $\tau_t^j \leq y_j$. Negative values of τ's are allowed (as transfers). Since both groups are of the same size, the government budget constraint is

$$\tau_t^A + \tau_t^B \leq 0.$$

Following the setting of policies, if $x_t = A$, group A has an opportunity to secede from this country, in which case each of its members receive an income of δy_A from t onward, where $\delta \in (0, 1)$ and the members of group B receive zero from t onward. If $x_t = B$, an analogous situation occurs where if group B secedes, all of its members receive an income of δy_B from t onward and the members of group A receive zero from t onward. Let $s_t^j = \{0, 1\}$ represent the secession decision which can be taken by group j if has the opportunity where 1 denotes secession. Note that only one group can receive a secession shock at a time.

The timing for the game is as follows:

- The group that was in power in the previous period starts out in power, and the secession shock x_t is realized.
- The group in power determines taxes and whether to transfer power to the other group.
- A secession decision is made if a group has the ability to secede.

1. Define the payoff-relevant state vector, the strategies and a Markov Perfect Equilibrium (MPE) in this game.
2. Show that if $\delta = 0$, there exists a unique MPE such that if $S_t = A$ then $\left(\tau_t^A = -y_B, \tau_t^B = y_B\right)$ and if $S_t = B$ then $\left(\tau_t^A = y_A, \tau_t^B = -y_A\right)$.
3. Explain why the above strategy profile may not be an equilibrium when $\delta > 0$.
4. Construct an equilibrium in which for $\delta \in \left(\underline{\delta}, \overline{\delta}\right)$, there exists an MPE in which whenever group j has the opportunity to secede, the political

regime switches to group j-dominance (unless it is already under group j-dominance). Explain why this equilibrium needs both parameter conditions $\delta > \underline{\delta}$ and $\delta < \overline{\delta}$.

5. How does this theory relate to and differs from other models of equilibrium institutional change?

6. How would to enrich this model to make it applicable more broadly to situations of within-country ethnic conflict?