

Complexity and Financial Panics

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Abstract

During extreme financial crises, all of a sudden, the financial world that was once rife with profit opportunities for financial institutions (banks, for short) becomes exceedingly complex. Confusion and uncertainty follow, ravaging financial markets and triggering massive flight-to-quality episodes. In this paper we propose a model of this phenomenon. In our model, banks normally collect information about their trading partners which assures them of the soundness of these relationships. However, when acute financial distress emerges in parts of the financial network, it is not enough to be informed about these partners, as it also becomes important to learn about the health of *their* trading partners. As conditions continue to deteriorate, banks must learn about the health of the trading partners of the trading partners of the trading partners, and so on. At some point, the cost of information gathering becomes too unmanageable for banks, uncertainty spikes, and they have no option but to withdraw from loan commitments and illiquid positions. A flight-to-quality ensues, and the financial crisis spreads.

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1 Introduction

The dramatic rise in investors' and banks' perceived uncertainty is at the core of the 2007-2009 U.S. financial crisis. All of a sudden, a financial world that was once rife with profit opportunities for financial institutions (banks, for short), was perceived to be exceedingly complex. Although the subprime shock was small relative to the financial institutions' capital, banks acted as if most of their counterparties were severely exposed to the shock (see Figure 1). Confusion and uncertainty followed, triggering the worst case of flight-to-quality that we have seen in the U.S. since the Great Depression.



Figure 1: The line corresponds to the TED spread in basis points (source: Bloomberg), the interest rate difference between the interbank loans (3 month LIBOR) and the US government debt (3 month Treasury bills). An increase in the TED spread typically reflects a higher perceived risk of default on interbank loans, that is, an increase in the banks' perceptions of counterparty risk.

In this paper we present a model of the sudden rise in complexity, followed by widespread panic in the financial sector. In the model, banks normally collect information about their direct trading partners which serves to assure them of the soundness of these relationships. However, when acute financial distress emerges in parts of the financial network, it is not enough to be informed about these partners, but it also becomes important for the banks to learn about the health of *their* trading partners. And as conditions continue to deteriorate, banks must learn about the health of the trading partners of the trading partners, of their trading partners, and so on. At some point, the cost of information gathering becomes too large and banks, now facing enormous uncertainty, choose to withdraw from loan commitments and illiquid positions. A flight-to-quality ensues, and the financial crisis spreads.

The starting point of our framework is a standard liquidity model where banks (representing financial institutions more broadly) have bilateral linkages in order to insure against local liquidity shocks. The whole financial system is a complex network of linkages which functions smoothly in the environments that it is designed to handle, even though no bank knows with certainty all the many possible connections within the network (that is, each bank knows the identities of the other banks but not their exposures). However, these linkages may also be the source of contagion when an unexpected event of financial distress arises somewhere in the network. Our point of departure with the literature is that we use this contagion mechanism *not* as the main object of study but as the source of *confusion* and *financial panic*. During normal times, banks only need to understand the financial health of their neighbors, which they can learn at low cost. In contrast, when a significant problem arises in parts of the network and the possibility of cascades arises, the number of nodes to be audited by each bank rises since it is possible that the shock may spread to the bank's counterparties. Eventually the problem becomes too complex for them to fully figure out, which means that banks now face significant uncertainty and they react to it by retrenching into liquidity-conservation mode.

This paper is related to several strands of literature. There is an extensive literature that highlights the possibility of network failures and contagion in financial markets. An incomplete list includes Allen and Gale (2000), Lagunoff and Schreft (2000), Rochet and Tirole (1996), Freixas, Parigi and Rochet (2000), Leitner (2005), Eisenberg and Noe (2001), Cifuentes, Ferucci and Shin (2005) (see Allen and Babus (2008) for a recent survey). These papers focus mainly on the mechanisms by which solvency and liquidity shocks may cascade through the financial network. In contrast, we take these phenomena as the reason for the rise in the *complexity* of the environment in which banks make their decisions, and focus on the effect of this complexity on banks' prudential actions. In this sense, our paper is related to the literature on flight-to-quality and Knightian uncertainty in financial markets, as in Caballero and Krishnamurthy (2008), Routledge and Zin (2004) and Easley and O'Hara (2005); and also to the related literature that investigates the effect of new events and innovations in financial markets, e.g. Liu, Pan, and Wang (2005), Brock and Manski (2008) and Simsek (2009). Our contribution relative to this literature is in endogenizing the rise in uncertainty from the behavior of the financial network itself. More broadly, this paper belongs to an extensive literature on flight-to-quality and financial crises that highlights the connection between panics and a decline in the financial system's ability to channel resources to the real economy (see, e.g., Caballero and Kurlat (2008), for a survey).

We build our argument in several steps. In Section 2 we first characterize the financial

network and describe a rare event as a perturbation to the structure of banks' shocks. Specifically, one bank suffers an unfamiliar liquidity shock for which it was unprepared. We next show that if banks can costlessly gather information about the network structure, the spreading of this shock into precautionary responses by other banks is typically contained. This scenario with no network uncertainty is the benchmark for our main results and is similar (although with an interior equilibrium) to Allen and Gale (2000).

Our main contribution is in Section 3, where we make information gathering costly. In this context, if the cascade is small, either because the liquidity shock is limited or because banks' buffers are significant, banks are able to gather the information they need about their indirect exposure to the liquidity shock and we are back to the full information results of Section 2. However, once cascades are large enough, banks are unable to collect the information they need to rule out a severe indirect hit. Their response to this uncertainty is to hoard liquidity and to retrench on their lending, which triggers a credit crunch. In Section 4 we show that under certain conditions, the response in Section 3 can be so extreme, that the entire financial system can collapse as a result of the flight to quality. The paper concludes with a final remarks section and several appendices.

2 The Environment and a Free-Information Benchmark

In this section we first introduce the environment and the characteristics of the financial network along with a shock which was unanticipated at the network formation stage (i.e. the financial network was not designed to deal with this shock). We next characterize the equilibrium for a benchmark case in which information gathering is free so that the market participants know the financial network.

2.1 The Environment

There are three dates $\{0, 1, 2\}$. There is a single good (one dollar) that serves as numeraire, which can be kept in liquid reserves or it can be loaned to production firms. If kept in liquid reserves, a unit of the good yields one unit in the next date. Instead, if a unit is loaned to firms at date 0, it then yields $R > 1$ units at date 2 if it is not unloaded before this date. At date 1, the lender can unload the loan (e.g. by settling it with the borrower at a discount) and receive $r < 1$ units. To simplify the notation, we assume $r \approx 0$ throughout this paper.

Banks and Their Liquidity Needs

The economy has $2n$ continuums of banks denoted by $\{b^j\}_{j=1}^{2n}$. Each of these continuums is composed of identical banks and, for simplicity, we refer to each continuum b^j as *bank* b^j , which is our unit of analysis.¹ Each bank b^j has initial assets which consist of y units of liquid reserves set aside for liquidity payments, $\bar{y}_0 \leq 1 - y$ units of *flexible reserves* set aside for making new loans at date 0 (but which can also be hoarded as liquid reserves) and $1 - y - \bar{y}_0$ units of loans. The bank's liabilities consist of a measure one of demand deposit contracts. A demand deposit contract pays $l_1 > 1$ at date 1 if the depositor is hit by a liquidity shock and $l_2 > l_1$ at date 2 if the depositor is not hit by a liquidity shock. Let $\omega^j \in [0, 1]$ be the measure of liquidity-driven depositors of bank b^j (i.e. the size of the liquidity shock experienced by the bank), which takes one of the three values in $\{\bar{\omega}, \omega_L, \omega_H\}$ with $\omega_H > \omega_L$ and $\bar{\omega} \equiv (\omega_H + \omega_L) / 2$, and suppose

$$y = l_1 \bar{\omega} \quad \text{and} \quad (1 - y) R = l_2 \bar{\omega}.$$

Note that, if the size of the liquidity shock is $\bar{\omega}$, the bank that loans all of its flexible reserves \bar{y}_0 at date 0 has assets just enough to pay l_1 (resp. l_2) to early (resp. late) depositors. The central trade-off in this economy will be whether the bank will loan its flexible reserves \bar{y}_0 (which it set aside for this purpose) or whether it will hoard some of this liquidity as a precautionary response to a rare event that we describe below.

The Financial Network

The liquidity needs at date 1 may not be evenly distributed among banks, which highlights one of the (many) reasons for an *interlinked financial network*. Moreover, the main source of complexity later on will be confusion about the linkages between different banks. To capture this possibility we let $i \in \{1, \dots, 2n\}$ denote *slots* in a financial network and consider a permutation $\rho : \{1, \dots, 2n\} \rightarrow \{1, \dots, 2n\}$ that assigns bank $b^{\rho(i)}$ to slot i . We consider a *financial network* denoted by:

$$\mathbf{b}(\rho) = (b^{\rho(1)} \rightarrow b^{\rho(2)} \rightarrow b^{\rho(3)} \rightarrow \dots \rightarrow b^{\rho(2n)} \rightarrow b^{\rho(1)}), \quad (1)$$

where the arc \rightarrow denotes that the bank in slot i (i.e., bank $b^{\rho(i)}$) has a demand deposit in the bank in the subsequent slot $i + 1$ (i.e., bank $b^{\rho(i+1)}$) equal to

$$z = (\bar{\omega} - \omega_L), \quad (2)$$

¹The only reason for the continuum is for banks to take other banks' decisions as given.

where we use modulo $2n$ arithmetic for the slot index i .² We refer to bank $b^{\rho(i+1)}$ as the *forward* neighbor of bank $b^{\rho(i)}$ (and similarly, to bank $b^{\rho(i)}$ as the *backward* neighbor of bank $b^{\rho(i+1)}$). The possibility of confusion arises later on from banks knowing the identity of other banks but not their particular linkages (i.e., the actual permutation ρ).

As we formally describe in Appendix A.1 (and similar to Allen-Gale (2000)), in the normal environment, the financial network facilitates liquidity insurance and enables liquidity to flow from banks that experience a low liquidity shock (ω_L) to the banks that experience a high liquidity shock (ω_H), even when the financial network $\mathbf{b}(\rho)$ is unknown to the banks. Our focus is on the effect of the financial interlinkages in case of an unanticipated shock for which the financial network is not necessarily designed for, which we describe next.

A Rare Event

At date 0 the banks learn that all banks will experience the average liquidity shock $\bar{\omega}$ at date 1, however, they also learn that one bank, $b^{\bar{j}}$, becomes distressed and loses $\theta \leq y$ of its liquid assets. As we formally demonstrate below, the losses in the distressed bank $b^{\bar{j}}$ might spill over to the other banks via the financial network $\mathbf{b}(\rho)$, thus the banks' knowledge of the financial network is potentially payoff relevant. In particular, this knowledge influences whether the banks use the flexible reserves \bar{y}_0 to make new loans or to hoard liquidity. We are thus lead to describe the central feature of our model: *the banks' uncertainty about the financial network $\mathbf{b}(\rho)$.*

Network Uncertainty and Auditing Technology

We let

$$\mathcal{B} = \{\mathbf{b}(\rho) \mid \rho : \{1, \dots, 2n\} \rightarrow \{1, \dots, 2n\} \text{ is a permutation}\}, \quad (3)$$

denote the set of possible financial networks. Each bank b^j observes its slot $i = \rho^{-1}(j)$ and the identities of the banks in its neighboring slots $i - 1$ and $i + 1$. This information narrows down the potential networks to the set:

$$\mathcal{B}^j(\rho) = \left\{ \mathbf{b}(\tilde{\rho}) \in \mathcal{B} \mid \begin{bmatrix} \tilde{\rho}(i-1) = \rho(i-1) \\ \tilde{\rho}(i) = \rho(i) \\ \tilde{\rho}(i+1) = \rho(i+1) \end{bmatrix}, \text{ where } i = \rho^{-1}(j) \right\}.$$

²In particular, i represents the slot with index $i' \in \{1, \dots, 2n\}$ that is the modulo $2n$ equivalent of integer i . For example, $i = 2n + 1$ represents the slot with index 1.

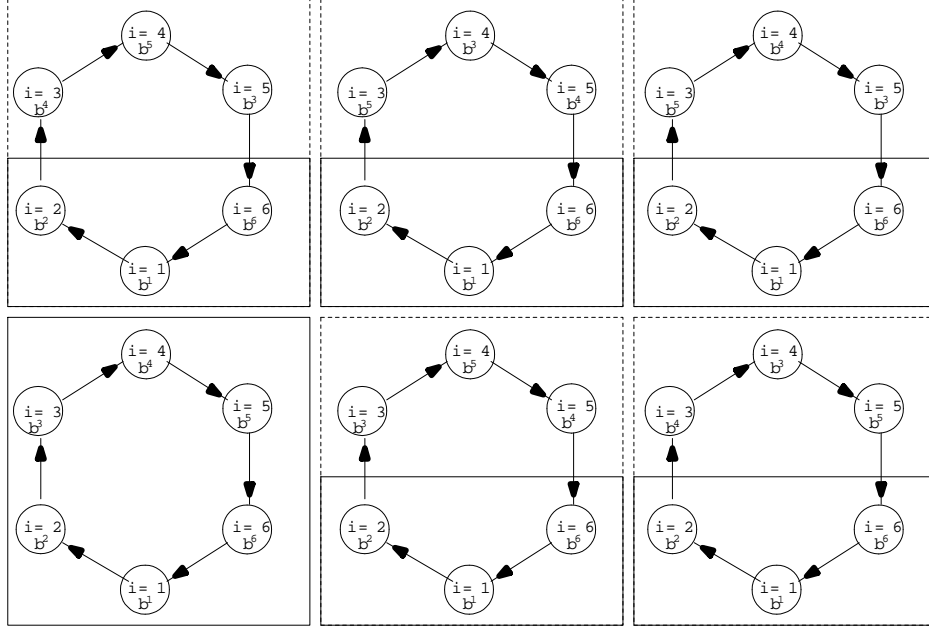


Figure 2: **The financial network and uncertainty.** The bottom-left box displays the actual financial network. Each circle corresponds to a slot in the financial network, and in this realization of the network, each slot i contains bank b^i (i.e. $\rho(i) = i$). The remaining boxes show the other networks that bank b^1 finds plausible after observing its neighbors (i.e. the set $\mathcal{B}^1(\rho)$). Bank b^1 cannot tell how the banks $\{b^3, b^4, b^5\}$ are ordered in slots $\{3, 4, 5\}$.

Note that the bank b^j does not know how the remaining banks $(b^j)_{j \notin \{\rho(i-1), \rho(i), \rho(i+1)\}}$ are assigned to the remaining slots (see Figure 2). In particular, each bank $b^j \neq \bar{b}^j$ knows that the bank \bar{b}^j is distressed, but it does not necessarily know the slot

$$\bar{i} = \rho^{-1}(\bar{j})$$

of the distressed bank. This is key, since it means that *a bank $b^j \neq \bar{b}^j$ does not necessarily know how far removed it is from the distressed bank.*

Each bank b^j can acquire more information about the financial network through an auditing technology. At date 0, a bank b^j in slot i (i.e. with $j = \rho(i)$) can exert effort to audit its forward neighbor $b^{\rho(i+1)}$ in order to learn the identity of this bank's forward neighbor $b^{\rho(i+2)}$. Continuing this way, a bank b^j that audits a number, a^j , of balance sheets learns the identity of its $a^j + 1$ forward neighbors and narrows the set of potential

financial networks to:

$$\mathcal{B}^j(\rho | a^j) = \left\{ \mathbf{b}(\tilde{\rho}) \in \mathcal{B} \mid \begin{bmatrix} \tilde{\rho}(i-1) = \rho(i-1) \\ \dots \\ \tilde{\rho}(i+a^j+1) = \rho(i+a^j+1) \end{bmatrix}, \text{ where } i = \rho^{-1}(j) \right\}.$$

We denote the posterior beliefs of bank b^j with $f^j(\cdot | \rho, a^j)$ which has support equal to $\mathcal{B}^j(\rho, a^j)$ given assumption:

Assumption (FS). Each bank has a prior belief $f^j(\cdot)$ over \mathcal{B} with full support.

In the example illustrated in Figure 2, if bank b^1 audits one balance sheet, then it would learn that bank b^3 is assigned to slot 3 and it would narrow down the set of networks to the two boxes at the left hand side of the bottom row in Figure 2.

Bank Preferences and Equilibrium

Consider a bank b^j and denote the bank's actual payments to early and late depositors by q_1^j and q_2^j (which may in principle be different than the contracted values l_1 and l_2). Because banks are infinitesimal, they make decisions taking the payments of the other banks as given. The bank makes the audit and liquidity hoarding decisions, $a^j \in \{0, 1, \dots, 2n-3\}$ and $y_0^j \in [0, \bar{y}_0]$, at date 0 (equivalently, $\bar{y}_0 - y_0^j \in [0, \bar{y}_0]$ denotes the number of new loans the bank makes at date 0). At date 1, the bank chooses to withdraw some of its deposits in the neighbor bank, which we denote by $z^j \in [0, z]$, and it may also unload some of its outstanding loans. The bank makes these decisions to maximize q_1^j until it can meet its liquidity obligations to depositors, that is, until $q_1^j = l_1$. Increasing q_1^j beyond l_1 has no benefit for the bank, thus once it satisfies its liquidity obligations, it then tries to maximize the return to the late depositors q_2^j .

We capture this behavior with the following objective function

$$v(\mathbf{1}\{q_1^j \leq l_1\} q_1^j + \mathbf{1}\{q_1^j \geq l_1\} q_2^j) - d(a^j), \quad (4)$$

where $v: \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ is a strictly concave and strictly increasing function and $d(\cdot)$ is an increasing and convex function which captures the bank's non-monetary disutility from auditing. When the bank b^j is making a decision that would lead to an uncertain outcome for (q_1^j, q_2^j) (which will be the case in Section 3), then it maximizes the expectation of the expression in (4) given its posterior beliefs $f^j(\cdot | \rho, a^j)$.

Suppose that the depositors' early/late liquidity shocks are observable, and a bank which is able to pay its late depositors at least l_1 at date 2 can refuse to pay the late

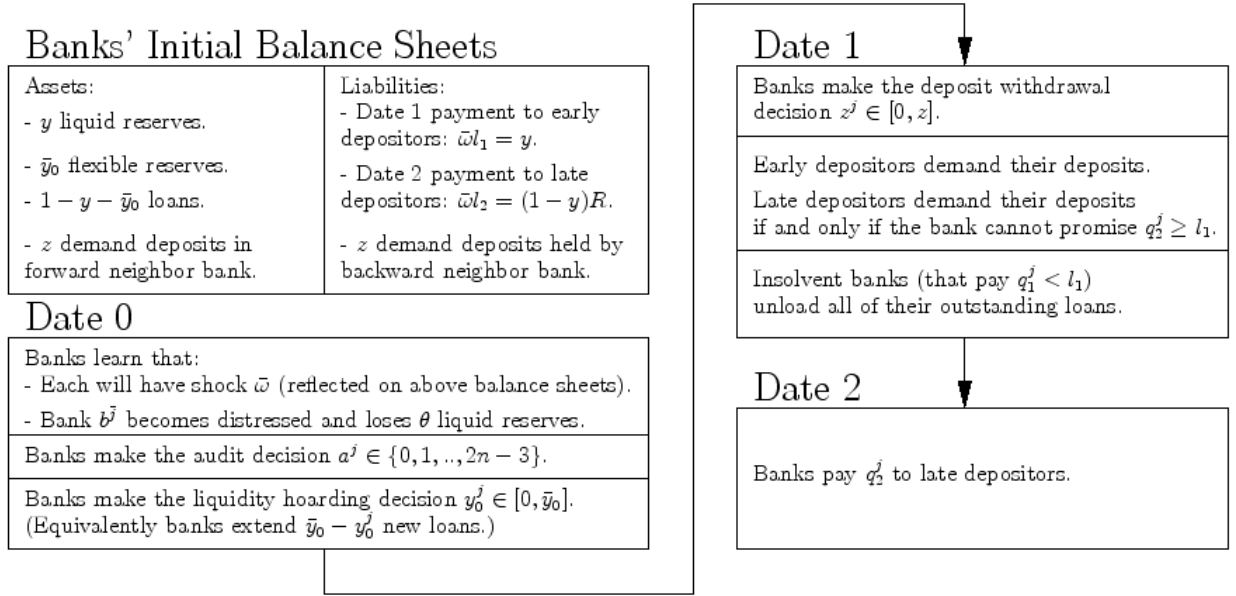


Figure 3: Timeline of events.

depositors if they arrive early.³ With this assumption, the continuation equilibrium for bank b^j at date 1 takes one of two forms. Either there is a *no-liquidation equilibrium* in which the bank is solvent and pays

$$q_1^j = l_1, q_2^j \geq l_1, \quad (5)$$

while the late depositors withdraw at date 2; or there is a *liquidation equilibrium* in which the bank is insolvent, unloads all outstanding loans, and pays

$$q_1^j < l_1, q_2^j = 0, \quad (6)$$

while all depositors (including the late depositors) draw their deposits at date 1.

Figure 3 recaps the timeline of events in this economy. We formally define the equilibrium as follows.

Definition 1. *The equilibrium is a collection of bank auditing, liquidity hoarding, deposit withdrawal, and payment decisions $\left[\{a^j(\rho), y_0^j(\rho), z^j(\rho), q_1^j(\rho), q_2^j(\rho)\}_j \right]_{\mathbf{b}(\rho) \in \mathcal{B}}$ such that, given the realization of the financial network $\mathbf{b}(\rho)$ and the rare event, each bank b^j maxi-*

³Without this assumption, there could be multiple equilibria for late depositors' early/late withdrawal decisions. In cases with multiple equilibria, this assumption selects the equilibrium in which no late depositor withdraws.

mizes expected utility in (4) according to its prior belief $f^j(\cdot)$ over \mathcal{B} , the insolvent banks (with $q_1^j(\rho) < l_1$) unload all of their outstanding loans at date 1 and the late depositors withdraw deposits early if and only if $q_2^j(\rho) < l_1$ (cf. Eqs. (5) and (6)).

We next turn to the characterization of equilibrium. Note that for each financial network $\mathbf{b}(\rho)$ and for each bank b^j , there exists a unique $k \in \{0, \dots, 2n - 1\}$ such that

$$j = \rho(\bar{i} - k),$$

which we define as the *distance* of bank b^j from the distressed bank. As we will see, the distance k will be the payoff relevant information for a bank b^j that decides how much liquidity to hoard at date 0 since it will determine whether or not the crisis that originates at the distressed bank $b^{\bar{i}}$ will cascade to bank b^j . The banks $b^{\rho(\bar{i}-1)}$, $b^{\rho(\bar{i})}$, $b^{\rho(\bar{i}+1)}$, respectively with distances 1, 0 and $2n - 1$, know their distances, but the remaining banks (with distances $k \in \{2, \dots, 2n - 2\}$) do not have this information a priori and they assign a positive probability to each $\tilde{k} \in \{2, \dots, 2n - 2\}$ (they rule out $\tilde{k} \in \{1, 2n - 1\}$ by observing their forward and backwards neighbors). Note, however, that the bank b^j can use the auditing technology to learn about the financial network and, in particular, about its distance from the distressed bank. A bank $b^{\rho(\bar{i}-k)}$ (with distance k) that audits $a^j \geq 1$ banks either learns its distance k (if $k \leq a^j + 1$) or it learns that $k \geq a^j + 2$.

In the remaining half of this section, we characterize the equilibrium in a benchmark case in which auditing is free so each bank learns its distance from the distressed bank. In subsequent sections, we characterize the equilibrium with costly audit and compare it with the free-information benchmark.

2.2 Free-Information Benchmark

We first describe a benchmark case in which auditing is free so each bank $b^{\rho(\bar{i}-k)}$ chooses full auditing $a^{\rho(\bar{i}-k)} = 2n - 3$. In this context banks learn the whole financial network $\mathbf{b}(\rho)$ and, in particular, their distances.

At date 0 all banks anticipate receiving a liquidity shock, $\bar{\omega}$, at date 1 and have liquid reserves equal to $y = \bar{\omega}l_1$ (plus \bar{y}_0 of flexible reserves), except for bank $b^{\bar{i}} = b^{\rho(\bar{i})}$ which has liquid reserves $y - \theta$. At date 1, the distressed bank $b^{\rho(\bar{i})}$ withdraws its deposits from the forward neighbor bank. As we show in Appendix A.2, this triggers further withdrawals until, in equilibrium, all cross deposits are withdrawn. That is

$$z^j = z \quad \forall j \in \{1, \dots, 2n\}. \quad (7)$$

In particular, bank $b^{\rho(\bar{t})}$ tries, but cannot, obtain any net liquidity through cross withdrawals. The bank also cannot obtain any liquidity by unloading the loans at date 1, since each unit of unloaded loan yields $r \approx 0$. Anticipating that it will not be able to obtain additional liquidity at date 1, the distressed bank $b^{\rho(\bar{t})}$ hoards some its flexible reserves \bar{y}_0 by cutting new loans at date 0 in order to meet its liquidity demand at date 1.

In order to promise *late* depositors *at least* l_1 , a bank with no liquid reserves left at the end of date 1 must have at least

$$1 - y - \bar{y}_0^n = \frac{(1 - \bar{\omega}) l_1}{R} \quad (8)$$

units of loans. The level \bar{y}_0^n is a *natural limit* on a bank's liquidity hoarding (which plans to deplete all of its liquidity at date 1) since any choice above this would make the bank necessarily insolvent. If the amount of flexible reserves \bar{y}_0 is greater than \bar{y}_0^n , then the bank can hoard at most \bar{y}_0^n of the flexible reserves while remaining solvent; or else it can hoard all of the flexible reserves \bar{y}_0 . Combining the two cases, a bank's buffer is given by

$$\beta = \min \{ \bar{y}_0, \bar{y}_0^n \}.$$

A bank can accommodate losses in liquid reserves up to the buffer β , but becomes insolvent when losses are beyond β . It follows that the distressed bank $b^{\rho(\bar{t})}$ will be insolvent whenever

$$\theta > \beta, \quad (9)$$

that is, whenever its losses in liquid reserves are greater than its buffer. Suppose this is the case so bank $b^{\rho(\bar{t})}$ is insolvent. Anticipating insolvency, this bank will hoard as much liquidity as it can $y_0^{\rho(\bar{t})} = \bar{y}_0$ (since it maximizes $q_1^{\rho(\bar{t})}$) and unloads all remaining loans at date 1. Since the bank is insolvent, all depositors (including late depositors) arrive early and the bank pays

$$q_1^{\rho(\bar{t})} = \frac{y + \bar{y}_0 - \theta + z q_1^{\rho(\bar{t}+1)}}{1 + z} < l_1, \quad (10)$$

where recall that $q_1^{\rho(\bar{t}+1)}$ denotes bank $b^{\rho(\bar{t}+1)}$'s payment to early depositors (which is equal to l_1 if bank $b^{\rho(\bar{t}+1)}$ is solvent).

Partial Cascades. Since bank $b^{\rho(\bar{t})}$ is insolvent, its backward neighbor bank $b^{\rho(\bar{t}-1)}$ will experience losses in its cross deposit holdings, which, if severe enough, may cause bank $b^{\rho(\bar{t}-1)}$'s insolvency. Once the crisis cascades to bank $b^{\rho(\bar{t}-1)}$, it may then similarly cascade to bank $b^{\rho(\bar{t}-2)}$, continuing its cascade through the network in this fashion.

We conjecture that, under appropriate parametric conditions, there exists a threshold $K \in \{1, \dots, 2n - 2\}$ such that all banks with distance $k \leq K - 1$ are insolvent (there are K such banks) while the banks with distance $k \geq K$ remain solvent. In other words, the crisis will partially cascade through the network but will be contained after $K \leq 2n - 2$ banks have failed. We refer to K as the *cascade size*.

Under this conjecture, bank $b^{\rho(\bar{\tau}+1)}$, which has a distance $2n - 1$, is solvent. Therefore $q_1^{\rho(\bar{\tau}+1)} = l_1$ and $q_1^{\rho(\bar{\tau})}$ in Eq. (10) can be calculated explicitly. Consider now the bank $b^{\rho(\bar{\tau}-1)}$ with distance 1 from the distressed bank. To remain solvent, this bank needs to pay l_1 on its deposits to bank $b^{\rho(\bar{\tau}-2)}$ but it receives only $q_1^{\rho(\bar{\tau})} < l_1$ on its deposits from the distressed bank $b^{\rho(\bar{\tau})}$, so it loses $z \left(l_1 - q_1^{\rho(\bar{\tau})} \right)$ in cross-deposits. Hence, bank $b^{\rho(\bar{\tau}-1)}$ will also go bankrupt if and only if its losses from cross-deposits are greater than its buffer, $z \left(l_1 - q_1^{\rho(\bar{\tau})} \right) > \beta$, which can be rewritten as

$$q_1^{\rho(\bar{\tau})} < l_1 - \frac{\beta}{z}. \quad (11)$$

If this condition fails, then the only insolvent bank is the original distressed bank and the cascade size is $K = 1$. If this condition holds, then bank $b^{\rho(\bar{\tau}-1)}$ anticipates insolvency, it will hoard as much liquidity as it can, i.e. $y_0^{\rho(\bar{\tau}-1)} = \bar{y}_0$ and it will pay all depositors

$$q_1^{\rho(\bar{\tau}-1)} = f \left(q_1^{\rho(\bar{\tau})} \right) \equiv \frac{y + \bar{y}_0 + zq_1^{\rho(\bar{\tau})}}{1 + z}. \quad (12)$$

From this point onwards, a pattern emerges. The payment by an insolvent bank $b^{\rho(i-k)}$ (with $k \geq 1$) is given by

$$q_1^{\rho(i-k)} = f \left(q_1^{\rho(i-(k-1))} \right)$$

and this bank's backward neighbor $b^{\rho(i-(k+1))}$ is also insolvent if and only if $q_1^{\rho(i-k)} < l_1 - \frac{\beta}{z}$. Hence, the payments of the insolvent banks converge to the fixed point of the function $f(\cdot)$ given by $y + \bar{y}_0$, and if⁴

$$y + \bar{y}_0 > l_1 - \frac{\beta}{z}, \quad (13)$$

⁴If condition (13) fails, then the sequence $\left(q_1^{\rho(i-k)} = f \left(q_1^{\rho(i-(k-1))} \right) \right)_k$ always remains below $l_1 - \frac{\beta}{z}$, and it can be checked that there is a *full cascade*, i.e. all banks are insolvent.

then (under Eq. (11)) there exists a unique $K \geq 2$ such that

$$q_1^{\rho(\bar{\tau}-k)} < l_1 - \frac{\beta}{z} \text{ for each } k \in \{0, \dots, K-2\} \quad (14)$$

and $q_1^{\rho(\bar{\tau}-(K-1))} \geq l_1 - \frac{\beta}{z}$.

If $2n - 2$ is greater than the solution, K , to this equation, i.e. if

$$2n - 2 \geq K, \quad (15)$$

then, Eq. (14) shows that (in addition to the trigger-distressed bank $b^{\rho(\bar{\tau})}$) all banks $b^{\rho(\bar{\tau}-k)}$ with distance $k \in \{1, \dots, K-1\}$ are insolvent since their losses from cross deposits are greater than their corresponding buffers. In contrast, bank $b^{\rho(\bar{\tau}-K)}$ (that receives $q_1^{\rho(\bar{\tau}-(K-1))}$ from its forward neighbor) is solvent, since it can meet its losses from cross deposits by hoarding some liquidity while still promising the late depositors at least l_1 (i.e. $q_2^{\rho(\bar{\tau}-K)} \geq l_1$). Since bank $b^{\rho(\bar{\tau}-K)}$ is solvent, all banks $b^{\rho(\bar{\tau}-k)}$ with distance $k \in \{K+1, \dots, 2n-1\}$ are also solvent since they do not incur losses in cross-deposits. Hence these banks do not hoard any liquidity, $y_0^{\rho(\bar{\tau}-k)} = 0$, and they pay $(q_1^{\rho(\bar{\tau}-k)} = l_1, q_2^{\rho(\bar{\tau}-k)} = l_2)$, verifying our conjecture for a partial cascade of size K under conditions (13) and (15).

Since our goal is to study the role of network uncertainty in generating a credit crunch, we take the partial cascades as the benchmark. The next proposition summarizes the above discussion and also characterizes the aggregate level of liquidity hoarding, which we use as a benchmark in subsequent sections.

Proposition 1. *Suppose the financial network is realized as $\mathbf{b}(\rho)$, auditing is free, and conditions (9), (13) and (15) hold. For a given financial network $\mathbf{b}(\rho)$, let $\bar{\tau} = \rho^{-1}(j)$ denote the slot of the distressed bank.*

(i) *For the continuation equilibrium (at date 1): The banks' equilibrium payments $(q_1^{\rho(\bar{\tau}-k)}, q_2^{\rho(\bar{\tau}-k)})$ are (weakly) increasing with respect to their distance k from the distressed bank and there is a partial cascade of size $K \leq 2n - 2$ where K is defined by Eq. (14). In particular, banks $\{b^{\rho(\bar{\tau}-k)}\}_{k=0}^{K-1}$ (with distance from the distressed bank $k \leq K - 1$) are insolvent while the remaining banks $\{b^{\rho(\bar{\tau}-k)}\}_{k=K}^{2n-1}$ (with distance $k \geq K$) are solvent.*

(ii) *For the ex-ante equilibrium (at date 0): Banks $\{b^{\rho(\bar{\tau}-k)}\}_{k=0}^{K-1}$ hoard as much liquidity as they can and unload all of their existing loans at date 1, while banks $\{b^{\rho(\bar{\tau}-k)}\}_{k=K+1}^{2n-1}$ do not hoard any liquidity or unload any loans. Bank $b^{\rho(\bar{\tau}-K)}$ hoards a level of liquidity $y_0^{\rho(\bar{\tau}-K)} = z \left(l_1 - q_1^{\rho(\bar{\tau}-(K-1))} \right)$ which is just enough to meet its losses from cross deposits (and does not unload any loans).*

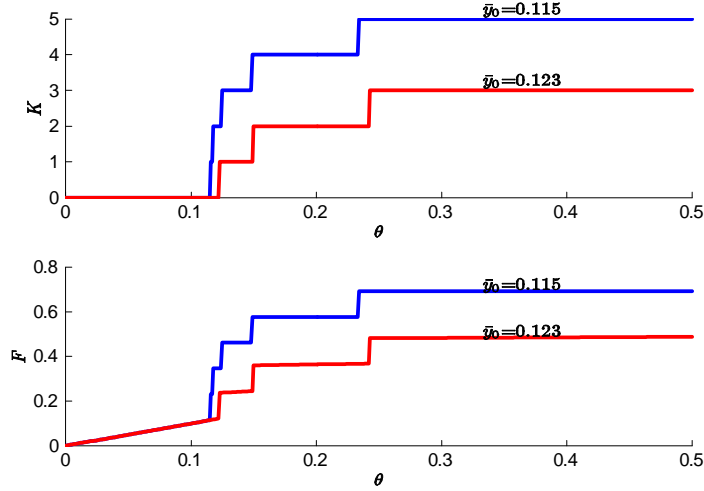


Figure 4: **The free-information benchmark.** The top figure plots the cascade size K as a function of the losses in the originating bank θ , for different levels of the flexible reserves \bar{y}_0 . The bottom figure plots the aggregate level of liquidity hoarding, \mathcal{F} , for the same set of $\{\bar{y}_0\}$.

The aggregate level of liquidity hoarding is:

$$\mathcal{F} \equiv \sum_j y_0^j = K \bar{y}_0 + y_0^{\rho(\bar{\tau}-K)}. \quad (16)$$

Discussion. Proposition 1 shows that, under appropriate parametric conditions, the equilibrium features a partial cascade and the aggregate level of liquidity hoarding, \mathcal{F} , is roughly linear in the size of the cascade K (and is roughly continuous in θ). Figure 4 demonstrates this result for particular parameterization of the model.

The top panel of the figure plots the cascade size K as a function of the losses in the originating bank θ for different levels of the flexible reserves \bar{y}_0 . This plot shows that the cascade size is increasing in the level of losses θ and decreasing in the level of flexible reserves \bar{y}_0 . Intuitively, with a higher θ and a lower \bar{y}_0 , there are more losses to be contained and the banks have less emergency reserves to counter these losses, thus increasing the spread of insolvency.

The bottom panel plots the aggregate level of liquidity hoarding \mathcal{F} , which is a measure of the severity of the credit crunch, as a function of θ . This plot shows that \mathcal{F} also increases with θ and falls with \bar{y}_0 . This is an intuitive result: In the free-information benchmark only the insolvent banks (and one transition bank) hoard liquidity, thus the more banks are insolvent (i.e. the greater K) the more liquidity is hoarded in the aggregate. Note

also that \mathcal{F} increases “smoothly” with θ .

These results offer a benchmark for the next sections. There we show that once auditing becomes costly, both K and \mathcal{F} may be non-monotonic in \bar{y}_0 and, more importantly, can jump with small increases in θ .

3 Endogenous Complexity and the Credit Crunch

We have now laid out the foundation for our main result. In this section we add the realistic assumption that auditing is costly and demonstrate that a massive credit crunch can arise in response to an endogenous increase in complexity once a bank in the network is sufficiently distressed. In other words, when K is large, it becomes too costly for banks to figure out their indirect exposure. This means that their perceived uncertainty rises and they eventually respond by hoarding liquidity as a precautionary measure (i.e., \mathcal{F} spikes).

Note that, unlike in Section 2.2, we cannot simplify the analysis by solving the equilibrium for a particular financial network $\mathbf{b}(\rho)$ in isolation, since, even when the realization of the financial network is $\mathbf{b}(\rho)$, each bank also assigns a positive probability to other financial networks $\mathbf{b}(\tilde{\rho}) \in \mathcal{B}$. As such, for a consistent analysis we must describe the equilibrium for any realization of the financial network $\mathbf{b}(\rho) \in \mathcal{B}$ (cf. Definition 1).

Solving this problem in full generality is cumbersome but we make assumptions on the form of the adjustment cost function, the banks’ objective function, and on the level of flexible reserves, that help simplify the exposition. First, we consider a convex and increasing cost function $d(\cdot)$ that satisfies

$$d(1) = 0 \quad \text{and} \quad d(2) > v(l_1 + l_2) - v(0). \quad (17)$$

This means that banks can audit one balance sheet for free but it is very costly to audit the second balance sheet. In particular, given the bank’s preferences in (4), the bank will never choose to audit the second balance sheet and thus each bank audits exactly one balance sheet, $\left[\{a^j(\rho) = 1\}_j\right]_{\mathbf{b}(\rho) \in \mathcal{B}}$. Given these audit decisions and the actual financial network $\mathbf{b}(\rho)$, a bank b^j has a posterior belief $f^j(\cdot|\rho,1)$ with support $\mathcal{B}^j(\rho,1)$, which is the set of financial networks in which the bank j knows the identities of its neighbors and its second forward neighbor. In particular, the bank $b^{\rho(\bar{\tau}-2)}$ learns its distance from the distressed bank $b^{\rho(\bar{\tau})}$ (in addition to banks $b^{\rho(\bar{\tau}-1)}$, $b^{\rho(\bar{\tau})}$, $b^{\rho(\bar{\tau}+1)}$ which already have this information from the outset). We denote the set of banks that know the slot of the

distressed bank (and thus their distance from this bank) by

$$B^{know}(\rho) = \{b^{\rho(\bar{i}-2)}, b^{\rho(\bar{i}-1)}, b^{\rho(\bar{i})}, b^{\rho(\bar{i}+1)}\}.$$

On the other hand, each bank $b^{\rho(\bar{i}-k)}$ with $k \in \{3, \dots, 2n-2\}$ learns that its distance is at least 3 (i.e. $\tilde{k} \geq 3$), but otherwise assigns a probability in $(0, 1)$ to all distances $\tilde{k} \in \{3, \dots, 2n-2\}$. We denote the set of banks that are uncertain about their distance by

$$B^{uncertain}(\rho) = \{b^{\rho(\bar{i}-3)}, b^{\rho(\bar{i}-4)}, \dots, b^{\rho(\bar{i}-(2n-2))}\}.$$

Second, we assume that the preference function $v(\cdot)$ in (4) is Leontieff $v(x) = (x^{1-\sigma} - 1) / (1 - \sigma)$ with $\sigma \rightarrow \infty$, so that the bank's objective is:

$$\min_{\mathbf{b}(\tilde{\rho}) \in \mathcal{B}^j(\rho, 1)} (\mathbf{1}\{q_1^j(\tilde{\rho}) \leq l_1\} q_1^j(\tilde{\rho}) + \mathbf{1}\{q_1^j(\tilde{\rho}) \geq l_1\} q_2^j(\tilde{\rho})) - d(a^j(\rho)). \quad (18)$$

This means that banks evaluate their decisions according to the worst possible network realization, $b(\tilde{\rho})$, which they find plausible.

The third and last assumption is that

$$\bar{y}_0 \leq \bar{y}_0^n. \quad (19)$$

That is, the bank has less flexible reserves than the natural limit on liquidity hoarding defined in Eq. (8) (which also implies that the buffer is given by $\beta = \bar{y}_0$). This condition ensures that, in the continuation equilibrium at date 1, the banks that have enough liquidity are also solvent (since, no matter how much of their flexible reserves they hoard, they have enough loans to pay the late depositors at least l_1 at date 2). We drop this condition in the next section.

We next turn to the characterization of the equilibrium under these simplifying assumptions. The banks make their liquidity hoarding decision at date 0 and deposit withdrawal decision at date 1 under uncertainty (before their date 1 losses from cross-deposits are realized). At date 1 the distressed bank $b^{\rho(\bar{i})}$ withdraws its deposits from the forward neighbor which leads to the withdrawal of all cross deposits (see Eq. (7) and Appendix A.2) as in the free-information benchmark. Thus, for any distressed bank, the only way to obtain additional liquidity at date 1 is through hoarding liquidity at date 0, which we characterize next.

A Sufficient Statistic for Liquidity Hoarding. Consider a bank $b^{\rho(\bar{z}-k)}$ other than the original distressed bank (i.e., $k > 0$). A *sufficient statistic* for this bank to make the liquidity hoarding decision is $q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho}) \leq l_1$, which is *the amount it receives in equilibrium from its forward neighbor*. In other words, to decide how much of its flexible reserves to hoard, this bank only needs to know whether (and how much) it will lose in cross-deposits. For example, if it knows with certainty that $q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho}) = l_1$ (i.e. its forward neighbor is solvent), then it hoards no liquidity, i.e. $y_0^{\rho(\bar{z}-k)} = 0$. If it knows with certainty that $q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho}) < l_1 - \beta/z$ (i.e. its forward neighbor will pay so little that this bank will also be insolvent), then it hoards as much liquidity as it can, i.e. $y_0^{\rho(\bar{z}-k)} = \bar{y}_0$. More generally, if the bank $b^{\rho(\bar{z}-k)}$ hoards some $y'_0 \in [0, \bar{y}_0]$ at date 0 and its forward neighbor pays $x \equiv q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho})$ at date 1, then this bank's payment can be written as

$$q_1^{\rho(\bar{z}-k)}(\tilde{\rho}) = q_1[y'_0, x] \quad \text{and} \quad q_2^{\rho(\bar{z}-k)}(\tilde{\rho}) = q_2[y'_0, x], \quad (20)$$

where the functions $q_1[y'_0, x]$ and $q_2[y'_0, x]$ are characterized in Eqs. (26) and (27) in Appendix A.2. At date 0, the bank does not necessarily know $x = q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho})$ and it has to choose the level of liquidity hoarding under uncertainty.

The characterization in Appendix A.2 also shows that $q_1[y'_0, x]$ and $q_2[y'_0, x]$ are (weakly) increasing in x for any given y'_0 . That is, the bank's payment is increasing in the amount it receives from its forward neighbor regardless of the ex-ante liquidity hoarding decision. Using this observation along with Eq. (20), the bank's objective value in (18) can be simplified and its optimization problem can be written as

$$\begin{aligned} & \max_{y'_0 \in [0, \bar{y}_0]} (\mathbf{1}\{q_1[y'_0, x^m] \geq l_1\} q_1[y'_0, x^m] + \mathbf{1}\{q_1[y'_0, x^m] < l_1\} q_2[y'_0, x^m]), \quad (21) \\ \text{s.t. } & x^m = \min \left\{ x \mid x = q_1^{\rho(\bar{z}-(k-1))}(\tilde{\rho}), \mathbf{b}(\tilde{\rho}) \in \mathcal{B}^j(\rho, 1) \right\}. \end{aligned}$$

In words, a bank $b^{\rho(\bar{z}-k)}$ (with $k > 0$) hoards liquidity *as if it will receive from its forward neighbor the lowest possible payment x^m* .

Distance Based and Monotonic Equilibrium. Next we define two equilibrium allocation notions that are useful for further characterization. First, we say that the equilibrium allocation is *distance based* if the bank's equilibrium payment can be written only as a function of its distance k from the distressed bank, that is, if there exists payment functions $Q_1, Q_2 : \{0, \dots, 2n-1\} \rightarrow \mathbb{R}$ such that

$$\left(q_1^{\rho(i-k)}(\rho), q_2^{\rho(i-k)}(\rho) \right) = (Q_1[k], Q_2[k])$$

for all $\mathbf{b}(\rho) \in \mathcal{B}$ and $k \in \{0, \dots, 2n - 1\}$. Second, we say that a distance based equilibrium is *monotonic* if the payment functions $Q_1[k], Q_2[k]$ are (weakly) increasing in k . In words, in a distance based and monotonic equilibrium, the banks that are further away from the distressed bank yield (weakly) higher payments.

We next conjecture that the equilibrium is distance based and monotonic (which we verify below). Then, a bank $b^{\rho(\bar{i}-k)}$'s uncertainty about the forward neighbor's payment $x = q_1^{\rho(\bar{i}-(k-1))}(\tilde{\rho}) = Q_1[k-1]$ reduces to its uncertainty about the forward neighbor's distance $k-1$, which is equal to one less than its own distance k . Hence, the problem in (21) can further be simplified by substituting $q_1^{\rho(i-(k-1))}(\tilde{\rho}) = Q_1[k-1]$. In particular, since a bank $b^{\rho(i-k)} \in B^{know}(\rho)$ (for $k > 0$) knows its distance k , it solves problem (21) with $x^m = Q_1[k-1]$.

On the other hand, a bank $b^{\rho(i-k)} \in B^{uncertain}(\rho)$ assigns a positive probability to all distances $\tilde{k} \in \{3, \dots, 2n-2\}$. Moreover, since the equilibrium is monotonic, its forward neighbor's payment $Q_1[\tilde{k}-1]$ is minimal for the distance $\tilde{k} = 3$, hence a bank $b^{\rho(i)} \in B^{uncertain}(\rho)$ solves problem (21) with $x^m = Q_1[2]$.

We are now in a position to state the main result of this section, which shows that *all banks that are uncertain about their distances to the distressed bank hoard liquidity as if they are closer to the distressed bank than they actually are.*

More specifically, all banks in $B^{uncertain}(\rho)$ hoard the level of liquidity that the bank with distance $\tilde{k} = 3$ would hoard in the free-information benchmark. When the cascade size is sufficiently large (i.e. $K \geq 3$) so that the bank with distance $\tilde{k} = 3$ in the free-information benchmark would hoard extensive liquidity, all banks in $B^{uncertain}(\rho)$ with actual distances $k > K$ also hoard large amounts of , even though they end up not needing it.

To state the result, we let $(y_{0,free}^j(\rho), q_{1,free}^j(\rho), q_{2,free}^j(\rho))_j$ denote the liquidity hoarding decisions and payments of banks in the free-information benchmark for each financial network $\mathbf{b}(\rho) \in \mathcal{B}$ (characterized in Proposition 1).

Proposition 2. *Suppose assumptions (FS), (17), and (18) are satisfied and conditions (9), (13), (15), and (19) hold. For a given financial network $\mathbf{b}(\rho)$, let $\bar{i} = \rho^{-1}(j)$ denote the slot of the distressed bank.*

(i) *For the continuation equilibrium (at date 1): The equilibrium allocation is distance based and monotonic. The cascade size in the continuation equilibrium is the same as in the free-information benchmark, that is, at date 1, banks $\{b^{\rho(\bar{i}-k)}\}_{k=0}^{K-1}$ are insolvent while banks $\{b^{\rho(\bar{i}-k)}\}_{k=K}^{2n-1}$ are solvent where K is defined in Eq. (14).*

(ii) *For the ex-ante equilibrium (at date 0): Each bank $b^j \in B^{know}(\rho)$ hoards the same*

level of liquidity $y_0^j(\rho) = y_{0,free}^j(\rho)$ as in the free-information benchmark, while each bank $b^j \in B^{uncertain}(\rho)$ hoards $y_0^j(\rho) = y_{0,free}^{\rho(\bar{i}-3)}(\rho)$, which is the level of liquidity bank $b^{\rho(\bar{i}-3)}$ would hoard in the free-information benchmark.

For the aggregate level of liquidity hoarding, there are three cases depending on the cascade size K :

If $K \leq 2$, then the crisis in the free-information benchmark would not cascade to bank $b^{\rho(\bar{i}-3)}$, which would hoard no liquidity, i.e. $y_{0,free}^{\rho(\bar{i}-3)}(\rho) = 0$. Thus, each bank $b^j \in B^{uncertain}(\rho)$ hoards no liquidity and the aggregate level of liquidity hoarding is equal to the benchmark Eq. (16).

If $K = 3$, then the crisis in the free-information benchmark would cascade to and stop at bank $b^{\rho(\bar{i}-3)}$, which would hoard an intermediate level of liquidity $y_{0,free}^{\rho(\bar{i}-3)}(\rho) \in [0, \bar{y}_0]$. Thus, each bank $b^j \in B^{uncertain}(\rho)$ hoards $y_{0,free}^{\rho(\bar{i}-3)}(\rho)$ and the aggregate level of liquidity hoarding is:

$$\mathcal{F} = \sum_j y_0^j = 3\bar{y}_0 + (2n - 4) y_{0,free}^{\rho(\bar{i}-3)}. \quad (22)$$

If $K \geq 4$, then in the free-information benchmark bank $b^{\rho(\bar{i}-3)}$ would be insolvent and would hoard as much liquidity as it can, i.e. $y_{0,free}^{\rho(\bar{i}-3)} = \bar{y}_0$. Thus, each bank $b^j \in B^{uncertain}(\rho)$ hoards as much liquidity as it can and the aggregate level of liquidity hoarding is:

$$\mathcal{F} = \sum_j y_0^j = (2n - 1) \bar{y}_0. \quad (23)$$

The proof of this result is relegated to Appendix A.2 since most of the intuition is provided in the discussion preceding the proposition. Among other features, the proof verifies that the equilibrium allocation at date 1 is distance based and monotonic, and that the cascade size is the same as in the free-information benchmark. The date 0 liquidity hoarding decisions are characterized as in part (ii) since the payments $Q_1[k-1]$ for $k \in \{1, 2, 2n-1\}$ (that a bank $b^{\rho(\bar{i}-k)} \in B^{know}(\rho)$ with $k > 0$ expects to receive) and the payment $Q_1[2]$ (that the banks in $B^{uncertain}(\rho)$ effectively expect to receive) are the same as their counterparts in the free-information benchmark.

Discussion. The plots in Figure 5 are the equivalent to those in the free-information case portrayed in Figure 4. The top panel plots the cascade size K as a function of the losses in the originating bank θ . The parameters satisfy condition (19) so that the cascade size in this case is the *same* as the cascade size in the free-information benchmark characterized in Proposition 1, and both figures coincide.

The key differences are in the bottom panel, which plots the aggregate level of liquidity

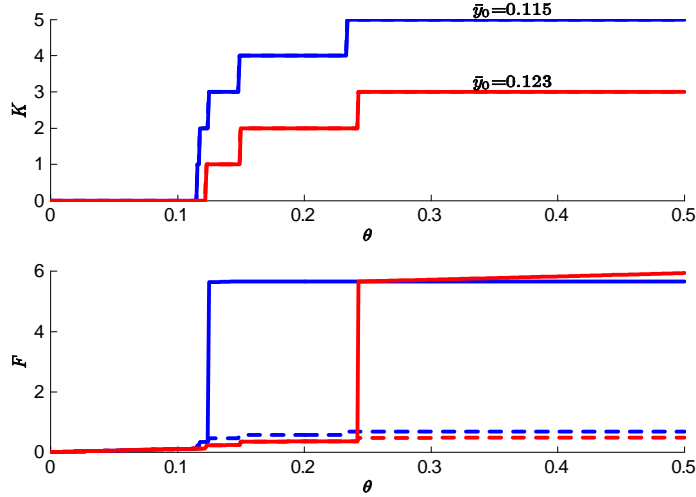


Figure 5: **The costly-audit equilibrium.** The top panel plots the cascade size K as a function of the losses in the originating bank θ for different levels of the flexible reserves \bar{y}_0 . The bottom panel plots aggregate level of liquidity hoarding \mathcal{F} for the same set of $\{\bar{y}_0\}$. The dashed lines in the bottom panel reproduce the free-information benchmark in Figure 4 for comparison.

hoarding \mathcal{F} as a function of θ . The solid lines correspond to the costly audit equilibrium characterized in Proposition 2, while the dashed lines reproduce the free-information benchmark also plotted in Figure 4. These plots demonstrate that, for low levels of K (i.e. for $K < 3$), the aggregate level of liquidity hoarding with costly-auditing is the same as the free-information benchmark, in particular, it increases roughly continuously with θ . As K switches from below 3 to above 3, the liquidity hoarding in the costly audit equilibrium make a very large and discontinuous jump. That is, when the losses (measuring the severity of the initial shock) are beyond a threshold, the cascade size becomes so large that banks are unable to tell whether they are connected to the distressed bank. All uncertain banks act as if they are closer to the distressed bank than they actually are, hoarding much more liquidity than in the free-information benchmark and leading to a severe credit crunch episode. This is our main result.

Note also that the aggregate level of liquidity hoarding (and the severity of the credit crunch) is not necessarily monotonic in the level of flexible reserves \bar{y}_0 . For example, when $\theta = 0.5$, Figure 5 shows that providing more flexibility to the banks by increasing \bar{y}_0 actually increases the level of aggregate liquidity hoarding. That is, at low levels of θ , an increase in flexibility stabilizes the system but the opposite may take place when the shock is sufficiently large. Intuitively, if the increase in flexibility is not sufficient

to contain the financial panic (by reducing the cascade size to manageable levels), more flexibility backfires since it enables banks to hoard more liquidity and therefore exacerbate the credit crunch.

4 The Collapse of the Financial System

Until now, the uncertainty that arises from endogenous complexity affects the extent of the credit crunch but not the number of banks that are insolvent, K . In this section we show that if banks have “too much” flexibility, in the sense that condition (19) no longer holds and

$$\bar{y}_0 \in (\bar{y}_0^n, 1 - y) \quad (24)$$

(which also implies $\beta = \bar{y}_0^n$), then the rise in uncertainty itself can increase the number of insolvent banks.

The reason is that a large precautionary liquidity hoarding compromises banks’ long run profitability by giving up high return R for low return 1. In this context, even if the worst outcome anticipated by a bank does not materialize, it may still become insolvent if sufficiently close (but farther than K) from the distressed bank. In other words, a bank’s large precautionary reaction improves its liquidation outcome when very close to the distressed bank but it does so at the cost of raising its vulnerability with respect to more benign scenarios. Since ex-post a large number of banks may find themselves in the latter situation, there can be a significant rise in the number of insolvencies as a result of the additional flexibility.

The analysis is very similar to that in the previous section. In particular, a bank’s payment still depends on its choice $y'_0 \in [0, \bar{y}_0]$ at date 0 and its forward neighbor’s payment $x \equiv q_1^{\rho(\bar{v}-(k-1))}(\tilde{\rho})$ at date 1. That is:

$$q_1^{\rho(\bar{v}-k)}(\tilde{\rho}) = q_1[y'_0, x] \quad \text{and} \quad q_2^{\rho(\bar{v}-k)}(\tilde{\rho}) = q_2[y'_0, x]$$

for some functions $q_1[y'_0, x]$ and $q_2[y'_0, x]$. However, the characterization of the piecewise functions $q_1[y'_0, x]$ and $q_2[y'_0, x]$ changes a little when condition (19) is not satisfied. In particular, these functions are identical to those in (26) and (27) in Appendix A.2 (as in Section 3) but now there is an additional insolvency region:

$$y'_0 > y_0^u [(l_1 - x) z].$$

The critical new element is the bound $y_0^u [(l_1 - x) z]$. This is a function of the losses from

cross-deposits and is calculated as *the level of liquidity hoarding above which the bank's loans and liquid reserves (net of losses) would not be sufficient to pay the late depositors at least l_1* . That is, $y_0^u [(l_1 - x) z]$ is the solution to

$$R(1 - y - y_0^u [(l_1 - x) z]) + y_0^u [(l_1 - x) z] - (l_1 - x) z = l_1(1 - \bar{\omega}).$$

We refer to scenarios where $y_0' > y_0^u [(l_1 - x) z]$ as, for lack of a better jargon, scenarios of *precautionary insolvency*.

The functions $q_1 [y_0', x]$ and $q_2 [y_0', x]$ remain (weakly) increasing in x . Moreover, we conjecture as before that the equilibrium is monotonic and distance based (which we verify below), so the banks' liquidity hoarding decisions still solve problem (21). It can be verified that all banks (except potentially bank $b^{\rho(\bar{\tau}+1)}$) hoard the level of liquidity characterized in part (ii) of Proposition 2. In particular, all banks $b^j \in B^{uncertain}(\rho)$ hoard the level of liquidity that the bank with distance $\tilde{k} = 3$ would hoard in the free-information benchmark. However, part (i) of the proposition, which characterizes the equilibrium at date 1, changes once \bar{y}_0 exceeds \bar{y}_0^n .

We divide the cases by the cascade size: $K \leq 2$, $K = 3$, and $K \geq 4$. In the first two of these cases there is no additional panic relative to the case where banks' flexibility is limited. If $K \leq 2$, each bank $b^j \in B^{uncertain}(\rho)$ hoards $y_0^j = 0$. The date 1 equilibrium in this case is as described in part (i) of Proposition 2, in particular, there are no precautionary insolvencies and the cascade size is equal to K . Similarly, if $K = 3$, each bank $b^j \in B^{uncertain}(\rho)$ hoards $y_0^j = y_{0,free}^{\rho(\bar{\tau}-3)} \leq \bar{y}_0^n$ (where the inequality follows since the transition bank $b^{\rho(\bar{\tau}-3)}$ is solvent in the free-information benchmark). Since $y_0^j \leq \bar{y}_0^n$, it can be seen that $y_0^j \leq y_0^u [(l_1 - x) z]$, so the banks in $B^{uncertain}(\rho)$ are solvent.⁵ It follows that there are no precautionary insolvencies and the equilibrium is again as described in part (i) of Proposition 2, with a cascade size equal to $K = 3$.

The new scenarios arise when $K \geq 4$. In this case, each bank $b^j \in B^{uncertain}(\rho)$ hoards $y_0^j = \bar{y}_0 > \bar{y}_0^n$, and may experience a precautionary insolvency depending on its losses from

⁵To see this, first note that $y_0^j \leq \bar{y}_0^n$, which implies

$$(R - 1) y_0^j \leq R \bar{y}_0^n - y_0^j = (1 - y) R - (1 - \bar{\omega}) l_1 - y_0^j,$$

where the equality follows from Eq. (8). Combining this inequality with the inequality $y_0^j \geq (l_1 - x) z$ (since $K \leq 3$, the banks in $B^{uncertain}(\rho)$ have sufficient liquid reserves at date 1) leads to

$$y_0^j \leq \frac{(1 - y) R - (1 - \bar{\omega}) l_1 - (l_1 - x) z}{R - 1} = y_0^u [(l_1 - x) z].$$

Note also that condition (19) implies $y_0^j \leq \bar{y}_0 \leq \bar{y}_0^n$ and thus rules out precautionary insolvencies by the above steps.

cross-deposits. To analyze this case, first note that the bound $y_0^u [(l_1 - x) z]$ is decreasing in $(l_1 - x) z$, and thus increasing in x . That is, the more a bank receives from its forward neighbor, the higher the bound above which it will experience a precautionary insolvency. Second, note the inequality, $y_0^u [0] < 1 - y$ (which follows from some algebra and using $l_2/l_1 < R$). Then, there are two subcases to consider depending on whether or not the level of flexible reserves \bar{y}_0 is greater than $y_0^u [0]$ (which is the highest value of the bound $y_0^u [(l_1 - x) z]$).

Subcase 1. If \bar{y}_0 is in the interval $(y_0^u [0], 1 - y]$, then \bar{y}_0 is always greater than the upper bound $y_0^u [(l_1 - x) z]$ and a bank b^j experiences a precautionary insolvency regardless of the amount x it receives from its forward neighbor. In particular, all banks in $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(2n-2))}\}$ are insolvent. It can be verified that the informed bank $b^{\rho(\bar{i}+1)}$ averts insolvency by hoarding some $y_0^{\rho(\bar{i}+1)} \leq \bar{y}_0^n$ (see Appendix A.2).

Subcase 2. If $\bar{y}_0 \in (\bar{y}_0^n, y_0^u [0])$, then there exists a unique $x[\bar{y}_0] \in (l_1 - \beta/z, l_1)$ that solves

$$y_0^u [(l_1 - x[\bar{y}_0]) z] = \bar{y}_0. \quad (25)$$

In this case, a bank b^j that hoards \bar{y}_0 of liquidity is insolvent if and only if it receives from its forward neighbor $x < x[\bar{y}_0]$ (so that $y_0^u [(l_1 - x) z] < \bar{y}_0$). By a similar analysis to that in Section 2.2 for the partial cascades (which we carry out in Appendix A.2), it can be checked that there exists $\hat{K} \in [K, 2n - 1]$ such that the banks $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(\hat{K}-1))}\}$ are insolvent while the banks $\{b^{\rho(\bar{i}-\hat{K})}, \dots, b^{\rho(\bar{i}-(2n-1))}\}$ are solvent. In other words, there is a partial cascade which is at least as large as (and potentially greater than) the partial cascade in the free-information benchmark.

We summarize our findings in the following proposition.⁶

Proposition 3. *Suppose assumptions (FS), (17) and (18) are satisfied and conditions (9), (13), (15) hold. Suppose also that condition (24) (which is the opposite of condition*

⁶Given the possibility of precautionary insolvencies, one may also wonder whether there could be multiple equilibria due to banks' coordination failures. Suppose, for example, $K = 3$, so that the crisis is contained after 3 banks fail. Could there also be a bad equilibrium in which all banks hoard the maximum level of liquidity, and their liquidity hoarding decisions are justified since their forward neighbors also hoard the maximum level of liquidity and experience a precautionary insolvency (thus paying a small q_1^j)?

This kind of coordination failure is not possible in our setup, precisely because of conditions (13) and (15). These conditions ensure that bank $b^{\rho(\bar{i}+1)}$ is always solvent, even if all other banks hoard the maximum level of liquidity and experience precautionary insolvencies. To see this, note that the losses from cross-deposits decrease as we move away from the distressed bank and eventually $q^{\rho(\bar{i}+2)} \geq l_1 - \beta/z$. Since bank $b^{\rho(\bar{i}+1)}$ expects to receive at least $l_1 - \beta/z$ from its forward neighbor, it can avoid insolvency by hoarding an intermediate level of liquidity. Hence, it is never optimal for bank $b^{\rho(\bar{i}+1)}$ to undergo a precautionary insolvency. But once we fix $q^{\rho(\bar{i}+1)} = l_1$, the rest of the equilibrium is uniquely determined as described above, that is, there is no coordination failure among banks.

(19)) holds. For a given financial network $\mathbf{b}(\rho)$, let $\bar{i} = \rho^{-1}(j)$ denote the slot of the distressed bank.

(i) For the ex-ante equilibrium (at date 0): Each bank $b^j \in \{b^{\rho(\bar{i})}, b^{\rho(\bar{i}-1)}, b^{\rho(\bar{i}-2)}\} \subset B^{\text{know}}(\rho)$ hoards the same level of liquidity $y_0^j(\rho) = y_{0,\text{free}}^j(\rho)$ that it would hoard in the free-information benchmark, while each bank $b^j \in B^{\text{uncertain}}(\rho)$ hoards $y_0^j(\rho) = y_{0,\text{free}}^{\rho(\bar{i}-3)}(\rho)$, which is the level of liquidity the bank $b^{\rho(\bar{i}-3)}$ would hoard in the free-information benchmark. The bank $b^j \in b^{\rho(\bar{i}+1)}$ hoards $y_0^{\rho(\bar{i}+1)}(\rho) \leq \bar{y}_0^n$ which is just enough to avert insolvency.

(ii) For the continuation equilibrium (at date 1): The equilibrium allocation is distance based and monotonic. There exists a unique $\hat{K} \in [K, 2n - 1]$ such that banks $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(\hat{K}-1))}\}$ are insolvent while banks $\{b^{\rho(\bar{i}-\hat{K})}, \dots, b^{\rho(\bar{i}-(2n-1))}\}$ are solvent. The cascade size \hat{K} is potentially larger than the cascade size K in the free-information benchmark. In particular, there are two cases:

If $K \leq 3$, then each bank $b^j \in B^{\text{uncertain}}(\rho)$ hoards some $y_0^j(\rho) \leq \bar{y}_0^n$, and avoids a precautionary insolvency. The cascade size in this case is identical to the free-information benchmark, i.e. $\hat{K} = K$.

If $K \geq 4$, then each bank $b^j \in B^{\text{uncertain}}(\rho)$ hoards $y_0^j(\rho) = \bar{y}_0 > \bar{y}_0^n$, which may lead to a precautionary insolvency. There are two sub-cases:

If $\bar{y}_0 \in (y_0^u [0], 1 - y]$, all banks $b^j \in B^{\text{uncertain}}(\rho)$ are insolvent and the cascade size is given by $\hat{K} = 2n - 1 > K \geq 4$.

If $\bar{y}_0 \in (\bar{y}_0^n, y_0^u [0])$, there exists a unique $x[\bar{y}_0] \in (l_1 - \beta/z, l_1)$ characterized by Eq. (25) such that bank $b^j \in B^{\text{uncertain}}(\rho)$ is insolvent if and only if its forward neighbor's payment is below $x[\bar{y}_0]$. The cascade size is an intermediate level $\hat{K} \in [K, 2n - 1]$.

Discussion. Figure 6 plots the cascade size \hat{K} as a function of θ , for different levels of the flexible reserves \bar{y}_0 . For comparison, the dashed lines plot the cascade size K in the free-information benchmark for the same parameters. The top panel corresponds to the case in which $\bar{y}_0 \leq \bar{y}_0^n$, i.e. when condition (19) holds. By Proposition 2, in this case there are no precautionary insolvencies and the cascade size is the same as the cascade size in the free-information benchmark. The second panel corresponds to a higher level of \bar{y}_0 that satisfies $\bar{y}_0 > \bar{y}_0^n$. In this case, precautionary insolvencies are possible, and for sufficiently large θ more banks are insolvent in the costly audit benchmark than in the free-information benchmark, i.e. $\hat{K} > K$. The third panel shows that, as we increase \bar{y}_0 , a sufficiently large shock θ may trigger a collapse of the whole financial system (i.e., $\hat{K} = 2n - 1$).

The bottom panel in Figure 6 shows that as \bar{y}_0 continues to rise, then at some point the

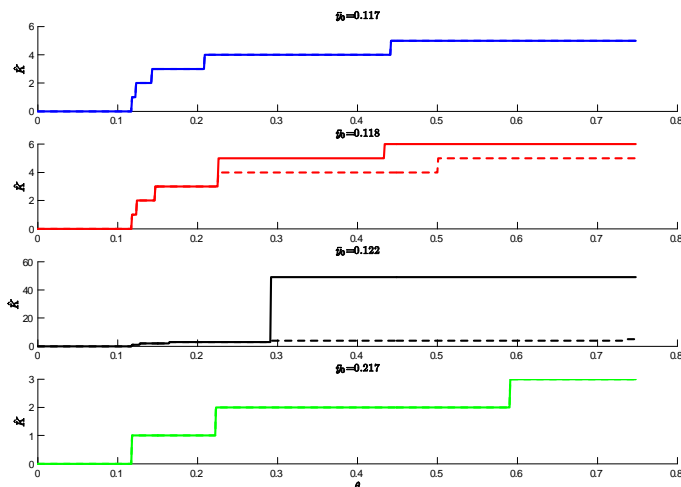


Figure 6: **The costly-audit equilibrium with precautionary insolvencies.** Each one of the four panels plots the cascade size \hat{K} as a function of θ for a different level of the flexible reserves \bar{y}_0 (in increasing order of \bar{y}_0 from top to bottom). The dashed lines plot the cascade size K in the free-information benchmark.

amplification disappears and again $\hat{K} = K$. That is, the effect of the flexible reserves \bar{y}_0 on the size of the cascade \hat{K} is non-monotonic: The whole financial system collapses with an intermediate level of \bar{y}_0 , but the health of the financial system is restored (and, in fact, is stronger) with sufficiently high levels of \bar{y}_0 . The intuition for this non-monotonicity is the same as the intuition for the non-monotonic effect of \bar{y}_0 on \mathcal{F} . Increasing the level of flexible reserves \bar{y}_0 reduces the cascade size K in the free-information benchmark. If this increase in flexibility is not sufficiently large, K does not fall to manageable levels and the financial panic remains. As long as there is a financial panic, the increase in \bar{y}_0 backfires and, in the current case, it also amplifies the cascade by generating more precautionary insolvencies. However, if the increase in \bar{y}_0 is sufficiently large, it may end the financial panic and restore the health of the financial system.

5 Final Remarks

Our model captures what appears to be a central feature of financial panics: During severe financial crises the complexity of the environment rises dramatically, and this in itself causes confusion and financial retrenchment. The perception of counterparty risk arises even in transactions among apparently sound financial institutions engaged in long

term relationships. All of a sudden, economic agents are faced with massive uncertainty as things are no longer business-as-usual. The collapse of Lehman Brothers during the current financial crisis is one such instance, which froze essentially all private credit markets and triggered massive run downs of credit lines and withdrawals even from the safest money market funds.

In the model we capture the complexity of the environment with the size of the partial cascades. When these cascades are small, banks only need to understand the financial health of their immediate neighbors to make their decisions. In contrast, when financial conditions worsen and cascades grow, banks need to understand and be informed about a larger share of the network. At some point, this is simply too costly and banks withdraw from intermediation rather than risk exposure to enormous uncertainty, which triggers a flight to quality.

We also showed that banks' flexibility, defined as their ability to hoard liquidity by not extending new loans or by selling illiquid assets while in distress, makes it harder for large cascades to develop, but if they do develop they can trigger more severe credit crunches and even a collapse in the financial system. Intuitively, a gain in flexibility is very useful if it succeeds in containing panic, but it can be counterproductive if it does not as it facilitates banks' withdrawal from intermediation.

An aspect we did not explore in this paper but one which we are currently pursuing in a related work, is that of secondary markets for loans at date 0. Our preliminary findings point to yet another amplification aspect of the mechanism we highlight in this paper: With full information, the distant banks (i.e., the banks with $k > K$) are the natural buyers of the loans sold by the distressed banks. However, once distant banks face uncertainty and become worried that they may be too close to the distressed bank, they cease to buy loans from these banks as they would rather hoard their liquidity, which exacerbates the network's distress.

There are some obvious policy conclusions that emerge from our framework. For example, there is clearly scope for having banks hold a larger buffer than they would be privately inclined to do. Also, transparency measures, by reducing the cost of gathering information, increase the resilience of the system to a lengthening in potential cascades. There is even an argument to limit banks' flexibility to cut loan commitments. However, we are interested in going beyond these observations, and in particular in exploring the impact of policies that modify the structure of the network. For example, there is an emerging consensus that the prevalence of bilateral OTC markets for CDS transactions compounded the confusion and complexity of the current financial crisis, and that it is imperative to organize these transactions in well capitalized exchanges to prevent

a recurrence. Our framework can help with the formal analysis of this type of policy considerations. We leave this analysis for future research.

A Appendix

A.1 The Normal Environment

The analysis in the text characterizes the equilibrium following a rare event for which the financial network is not prepared. In this Appendix, we describe the functioning of the financial network in the normal environment. In particular, we show that the financial network enables the banks to insure against heterogeneous liquidity shocks and facilitates the flow of liquidity across banks, even if the banks are uncertain about the network structure.

In the normal environment, there are three aggregate states of the world, denoted by $s(0)$, $s(r)$ and $s(g)$, revealed at date 0. In state $s(0)$ all banks expect to receive at date 1 the same liquidity shock $\bar{\omega}$. The states $s(r)$ and $s(g)$ are realized with equal probability and the liquidity shocks in these states are heterogeneous across banks. More specifically, the banks are divided half and half between two types: *red* and *green*. In state $s(r)$ (resp. $s(g)$), the banks with red type (resp. green type) expect to receive a high liquidity shock, ω_H , and the other banks expect to receive a low liquidity shock, ω_L . This means that in states $s(r)$ and $s(g)$ there is enough aggregate liquidity but there is a need to transfer liquidity across banks.

To transfer liquidity in states $s(r)$ and $s(g)$, the banks form the financial network of bilateral deposits in (1). We say that the financial network is *consistent* if all odd slots (resp. all even slots) contain banks of the same type, which means that red and green type banks alternate around the financial circle. We restrict the set of feasible networks to consistent ones (as opposed to, for example, any circular network in which banks may be arbitrarily ordered around the circle), since these networks ensure that each bank that needs liquidity has deposits on a bank with excess liquidity, facilitating bilateral liquidity insurance (see below) with the minimally required level of cross-deposits z in (2).

Banks' types and the financial network are realized as follows: First the types of banks are realized at random (half of the banks become red type and the other half green type); then a particular consistent financial network $\mathbf{b}(\rho)$ (with respect to these types) is realized. Banks' types are their private information, thus the set \mathcal{B} in (3) represents the set of consistent financial networks from an ex-ante point of view (i.e. before the types of the banks are realized). This shock structure ensures that the actual realization of the

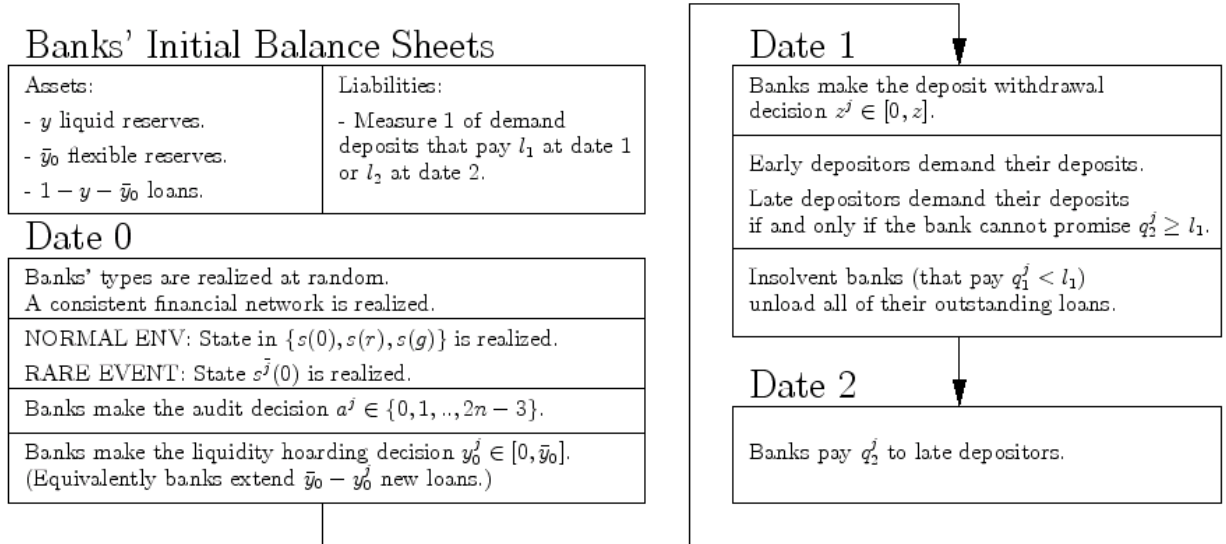


Figure 7: Timeline of events, for both the normal environment and the rare event.

financial network is always consistent while the banks' uncertainty about the financial network is still described as in the rare event case. In particular, a bank b^j observes the slots of its neighbors (and since the network is consistent, it also indirectly learns the types of its neighbors), but it does not know the slots (or the types) of the remaining banks $(b^j)_{j \notin \{\rho(i-1), \rho(i), \rho(i+1)\}}$ (see Figure 2). By auditing a^j balance sheets, the bank can further narrow down the set of possible networks to $\mathcal{B}^j(\rho | a^j)$.

Figure 7 recaps the timeline of events in this economy both for the normal environment and the rare event analyzed in the main text. Note that the rare event analyzed in the main text is characterized by an unanticipated aggregate state $s^{\bar{j}}(0)$ which is very similar to one of the states in the normal environment (i.e. the state $s(0)$) except for the fact that one bank, $b^{\bar{j}}$, becomes distressed and loses θ of its liquid reserves.

The equilibrium in the normal environment is a collection of bank auditing, liquidity hoarding, deposit withdrawal, and payment decisions $\left[\{a^j(\rho), y_0^j(\rho), z^j(\rho), q_1^j(\rho), q_2^j(\rho)\}_j \right]_{\mathbf{b}(\rho) \in \mathcal{B}}$ such that, for each realization of the financial network $\mathbf{b}(\rho)$ and the aggregate state in $\{s(0), s(r), s(g)\}$, each bank b^j makes decisions that maximize the preferences in (4), and the late depositors withdraw deposits early if and only if $q_2^j < l_1$ (cf. Eqs. (5) and (6)). We next characterize this equilibrium.

The Normal Functioning of the Financial Network. We claim that, in the normal environment, the financial network facilitates liquidity flow and enables each bank b^j to pay the contracted values $(q_1^j = l_1, q_2^j = l_2)$ in each state of the world. Suppose that a

consistent financial network, $\mathbf{b}(\rho)$ and state $s(r)$ is realized, and suppose without loss of generality that red type banks are assigned to odd slots (the case in which red type banks are assigned to even slots is symmetric). It suffices to prove the statement for this case since the case in which $s(g)$ is realized is symmetric to the $s(r)$ case, and the case in which $s(0)$ is realized is trivial.

We conjecture (and verify below) that each bank b^j chooses not to audit (for any positive audit costs $d(\cdot) > 0$) and hoards no liquidity, i.e. $a^j = 0$ and $y_0^j = 0$. Consider the equilibrium at date 1. A red type bank, $b^{\rho(2i-1)}$, (which is assigned to an odd slot by assumption) needs liquidity so it draws its deposits from the forward neighbor bank, i.e. chooses $z^{\rho(2i-1)} = z$. For each green type bank, $b^{\rho(2i)}$, regardless of the financial network in $\mathcal{B}^{\rho(2i)}(\rho)$, drawing $z^{\rho(2i)} \in [0, z]$ deposits leads to the payments $q^{\rho(2i)} = l_1$ and

$$q_2^{\rho(2i)} = \frac{(1-y)R + z^{\rho(2i)}l_1 + (z - z^{\rho(2i)})l_2}{1 - \omega_L}.$$

Since $l_2 > l_1$ and the preferences are given by (4), the green type banks do not draw their deposits regardless of their beliefs $f^{\rho(2i)}(\cdot | \rho)$, i.e. they choose $z^{\rho(2i)} = 0$. It follows that liquidity flows through the network at date 1 even though each bank is uncertain about the network structure. In particular, each bank b^j pays the contracted values ($q_1^j = l_1, q_2^j = l_2$) in state $s(r)$ (and similarly in states $s(g)$ and $s(0)$).

We next consider the equilibrium at date 0 and verify our conjecture that the banks choose not to audit and not to hoard any liquidity. First note that a bank $b^{\rho(i)}$ in need of liquidity at date 1 is able to obtain it by withdrawing its deposits in the forward neighbor at a cost of l_2/l_1 units at date 2 for each unit of liquidity. The bank could also obtain liquidity by hoarding flexible reserves at date 0 but this would cost $R > l_2/l_1$ units for each unit of liquidity (since $R > l_2 > l_1 > 1$). Therefore, each bank $b^{\rho(i)}$ optimally chooses not to hoard any liquidity at date 0. Second note that a bank's, $b^{\rho(i)}$, optimal actions (for liquidity hoarding at date 0 and deposit withdrawal at date 1) only depend on its slot i (and only on its parity), and in particular, it is independent of the financial network in $\mathcal{B}^{\rho(i)}(\rho)$. Thus the bank does not benefit from auditing and optimally chooses not to audit, $a^{\rho(i)} = 0$ (whenever $d(\cdot) > 0$), thus verifying our conjecture. This completes the proof of our claim that, in the normal environment, the financial network facilitates liquidity flow across banks and enables each bank b^j to pay the contracted values ($q_1^j = l_1, q_2^j = l_2$) in each aggregate state.

A.2 Proofs Omitted in the Main Text

Proof of Eq. (7) for Sections 2.2 and 3. We claim that all cross-deposits are fully withdrawn, i.e. Eq. (7) holds, in both the free-information benchmark analyzed in Section 2.2 and the costly audit model analyzed in Section 3. By condition (9), the original distressed bank, $b^{\rho(\bar{i})}$, is insolvent thus it withdraws all of its deposits, i.e. $z^{\rho(\bar{i})} = z$. Suppose that, for some $k \in \{0, \dots, 2n - 1\}$, bank $b^{\rho(\bar{i}-(k+1))}$ withdraws all of its deposits in bank $b^{\rho(\bar{i}-k)}$. We claim that bank $b^{\rho(\bar{i}-k)}$ also withdraws deposits, i.e. $z^{\rho(\bar{i}-k)} = z$, which proves Eq. (7) by induction.

To prove the claim, we first consider the free-information benchmark and analyze two cases in turn. As the first case, suppose that the forward neighbor of bank $b^{\rho(\bar{i}-k)}$ is insolvent (i.e. it pays $q_1^{\rho(i-(k-1))} < l_1$ and $q_2^{\rho(i-(k-1))} = 0$). Recall that bank $b^{\rho(\bar{i}-k)}$ is small and takes the payment of its forward neighbor as given (see footnote 1), in particular, it cannot potentially bail out its forward neighbor by withdrawing less than z . This further implies that the bank withdraws all of its deposits from its forward neighbor, i.e. $z^{\rho(\bar{i}-k)} = z$. As the second case, suppose that the forward neighbor bank, $b^{\rho(\bar{i}-(k-1))}$, is solvent, i.e. $q_1^{\rho(\bar{i}-(k-1))} = l_1$. In this case, bank $b^{\rho(\bar{i}-k)}$ needs liquidity z (to pay its backward neighbor) and it can obtain this liquidity either by withdrawing its deposits, which costs l_2/l_1 units at date 2 per unit of liquidity, or by hoarding flexible reserves at date 0, which costs $R > l_2/l_1$ units per unit of liquidity. Since the former is a cheaper way to obtain liquidity, bank $b^{\rho(\bar{i}-k)}$ withdraws all of its deposits from its forward neighbor, proving our claim that $z^{\rho(\bar{i}-k)} = z$.

Next consider the costly audit model of Section 3. Recall that bank $b^{\rho(\bar{i}-k)}$ makes the deposit withdrawal decision before the resolution of uncertainty for cross-deposits (see Figure 3). As the first case, suppose that bank $b^{\rho(\bar{i}-k)}$ assigns a positive probability to a network structure $\mathbf{b}(\tilde{\rho})$ such that $q_1^{\rho(i-(k-1))}(\tilde{\rho}) < l_1$ (that is, suppose the bank assigns a positive probability that its forward neighbor will be insolvent). Since the bank takes the payment of its forward neighbor as given and its preferences are given by the Leontieff form in (18), in this case the bank necessarily withdraws all of its deposits, i.e. $z^{\rho(\bar{i}-k)} = z$. Next suppose bank $b^{\rho(\bar{i}-k)}$ believes that $q_1^{\rho(i-(k-1))}(\tilde{\rho}) = l_1$ with probability 1 (that is, the bank knows that its forward neighbor is solvent). In this case, as in the free-information benchmark, the bank withdraws $z^{\rho(\bar{i}-k)} = z$ to meet its liquidity obligations to its backward neighbor. This completes the proof of the claim and proves Eq. (7) by induction.

Proof of Proposition 1. Contained in the discussion preceding the proposition.

Characterization of Banks' Payment Functions $q_1 [y'_0, x]$ and $q_2 [y'_0, x]$ in Section

3. If bank $b^{\rho(\bar{i}-k)}$ hoards a level of liquidity $y'_0 \in [0, \bar{y}_0]$ at date 0, and its forward neighbor pays $x = q_1^{\rho(\bar{i}-(k-1))}(\tilde{\rho})$ at date 1 (and if condition (19) holds), then this bank's payment is given by functions $q_1 [y'_0, x]$ and $q_2 [y'_0, x]$ which are characterized as follows:

Case 1. If $x \in [l_1 - \beta/z, l_1]$ and $y'_0 \geq (l_1 - x)z$, then

$$q_1 = l_1 \text{ and } q_2 = \frac{y'_0 - (l_1 - x)z + (1 - y - y'_0)R}{1 - \bar{\omega}} \geq l_1. \quad (26)$$

Case 2. If $x < l_1 - \beta/z$ or $y'_0 < (l_1 - x)z$, then

$$q_1 = \frac{y + y'_0 + zx}{1 + z} \leq l_1 \text{ and } q_2 = 0. \quad (27)$$

The first case characterizes the payment when the bank's losses from cross-deposits do not exceed its buffer and the bank has hoarded enough flexible reserves to counter these losses. In this case, the bank is solvent and pays according to (26). The second case characterizes the payment when the bank's losses from cross-deposits exceed its buffer, or when the losses do not exceed the buffer but the bank has not hoarded enough flexible reserves to counter these losses. In this case, the bank is insolvent and pays according to (27).

Proof of Proposition 2. First consider part (i) taking as given the characterization of the liquidity hoarding decisions in part (ii). Note that the liquidity hoarding decision of each bank depends only on its distance from the distressed bank, which implies that the payments of banks in the continuation equilibrium can be written as a function of their distances, i.e. that the equilibrium is distance based. The characterization in part (ii) shows that each bank $b^{\rho(\bar{i}-k)} \in \{b^{\rho(\bar{i})}, b^{\rho(\bar{i}-1)}, \dots, b^{\rho(\bar{i}-(K-1))}\}$ that would be insolvent in the free-information benchmark hoards $y_0^j = \bar{y}_0$, and thus it pays the same allocation it would pay in the free-information benchmark:

$$Q_1 [k] \equiv q_{1,free}^{\rho(\bar{i}-k)}(\rho) \leq l_1 \text{ and } Q_2 [k] \equiv q_{2,free}^{\rho(\bar{i}-k)}(\rho) = 0 \text{ for } k \in \{0, \dots, K-1\}. \quad (28)$$

The bank $b^{\rho(\bar{i}-K)}$ hoards at least as much liquidity as it would hoard in the free-information benchmark, thus it is solvent given condition (19) (which ensures that hoarding too much liquidity does not cause insolvency) and pays (cf. Eq. (26)):

$$Q_1 [K] = l_1 \text{ and } Q_2 [K] \geq l_1. \quad (29)$$

The banks $b^{\rho(\bar{i}-k)} \in \{b^{\rho(\bar{i}-(K+1))}, b^{\rho(\bar{i}-(K+2))}, \dots, b^{\rho(\bar{i}-(2n-1))}\}$ are solvent and thus pay (cf. Eq. (26)):

$$Q_1[k] = l_1 \text{ and } Q_2[k] = \frac{y_0^{\rho(\bar{i}-k)} + \left(1 - y - y_0^{\rho(\bar{i}-k)}\right) R}{1 - \bar{\omega}} \geq l_1, \text{ for } k \in \{K + 1, \dots, 2n - 1\}. \quad (30)$$

In particular, the size of the cascade is K as it is in the free-information benchmark. Since $q_{1,free}^{\rho(i-k)}(\rho)$ is increasing in k (see Proposition 1) and $y_0^{\rho(\bar{i}-k)}$ is decreasing in k (given the liquidity hoarding decisions in part (ii)), the characterization in (28) through (30) also implies that the payments, $Q_1[k]$ and $Q_2[k]$, are increasing in k and proves that the distance based equilibrium is monotonic.

We next turn to the liquidity hoarding decisions at date 0 and prove that the choices prescribed in part (ii) are optimal. Consider first the banks in $B^{know}(\rho)$. Comparing the characterization of the continuation equilibrium in (28) through (30) to the characterization in Proposition 1, each bank $b^j \in B^{know}(\rho)$ expects to receive the same payment from its forward neighbor compared to what it would receive in the free-information benchmark (i.e. each bank $b^j \in B^{know}(\rho)$ solves problem (21) with $x^m = q_{1,free}^{\rho(\bar{i}-(k-1))}$). Thus it also hoards the same level of liquidity that it would hoard in the free-information benchmark.

Next we consider a bank $b^j \in B^{uncertain}(\rho)$ which solves problem (21) with $x^m = Q_1[2]$. We claim that $Q_1[2]$ characterized in Eqs. (28) through (30) is equal to $q_{1,free}^{\rho(2)}$ (the payment of the forward neighbor of bank $b^{\rho(\bar{i}-3)}$ in the free-information benchmark), which in turn proves that the bank b^j hoards the same level of liquidity $y_{0,free}^{\rho(\bar{i}-3)}$ that $b^{\rho(\bar{i}-3)}$ would hoard in the free-information benchmark. To prove the claim that $Q_1[2] = q_{1,free}^{\rho(2)}$, first suppose that $K \leq 2$. Note that in this case $Q_1[2]$ is given by Eq. (29) or Eq. (30) and in either case $Q_1[2] = l_1$. Note that by Proposition 1, $q_{1,free}^{\rho(2)} = l_1$ when $K \leq 2$, proving the claim in this case. Next suppose $K \geq 3$ and note that in this case $Q_1[2]$ is given by Eq. (28) which shows $Q_1[2] = q_{1,free}^{\rho(\bar{i}-2)}(\rho)$, completing the proof of part (ii).

The characterization for the aggregate level of liquidity hoarding for the cases $K \leq 2$, $K = 3$ and $K \geq 4$ then trivially follow from part (ii) and Proposition 1, thus completing the proof.

Proof of Proposition 3. Most of the proof is contained in the discussion preceding the proposition. Here, we consider in turn the subcases 1 and 2 (for case $K \geq 4$) and we verify the claims in the main text. We also verify the conjecture that the equilibrium is distance based and monotonic.

Subcase 1. If $\bar{y}_0 \in (y_0^u [0], 1 - y]$, then

$$\bar{y}_0 > y_0^u [0] \geq y_0^u [(l_1 - x) z]$$

for any $x \in [0, l_1]$. This implies that all banks in $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(2n-2))}\}$ are insolvent since they hoard a level of liquidity greater than their corresponding upper limits. These banks' payments are characterized by the system of equations

$$q_1^{\rho(\bar{i}-k)} = f\left(q_1^{\rho(\bar{i}-(k-1))}\right) \text{ for each } k \in \{1, \dots, 2n-2\}, \quad (31)$$

where $f(\cdot)$ is defined in Eq. (12) and the initial condition $q_1^{\rho(\bar{i})}$ is given by Eq. (10) (after plugging in $q_1^{\rho(\bar{i}+1)} = l_1$).

By condition (13), the solution to the above system is increasing (and converges to the fixed point $y + \bar{y}_0 \leq 1 < l_1$), verifying our conjecture that the equilibrium is distance based and monotonic. By condition (15), we have $K \leq 2n-2$, which implies $q_1^{\rho(\bar{i}-(2n-2))} > q_1^{\rho(\bar{i}-(K-1))} = q_{1,free}^{\rho(\bar{i}-(K-1))}$. Then, since bank $b^{\rho(\bar{i}-K)}$ in the free-information benchmark is able to avert insolvency by hoarding the level $y_{0,free}^{\rho(\bar{i}-K)} \leq \bar{y}_0^n$, the informed bank $b^{\rho(\bar{i}+1)}$ in this case can also avert insolvency by hoarding $y_0^{\rho(\bar{i}+1)} \leq \bar{y}_0^n$. It follows that the cascade size is $\hat{K} = 2n - 1$, which is greater than the free-information cascade size K (under condition (15)), completing the characterization of the date 1 equilibrium in this case.

Subcase 2. If $\bar{y}_0 \in (\bar{y}_0^n, y_0^u [0])$, there exists a unique $x[\bar{y}_0] \in (l_1 - \bar{y}_0^n/z, l_1)$, characterized in Eq. (25) and increasing in \bar{y}_0 , such that a bank $b^j \in B^{uncertain}(\rho)$ is insolvent if and only if receives from its forward neighbor $x < x[\bar{y}_0]$. Using the conjecture that the equilibrium is distance based and monotonic, we further conjecture that the banks $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(\hat{K}-1))}\}$ are insolvent while the banks $\{b^{\rho(\bar{i}-\hat{K})}, \dots, b^{\rho(\bar{i}-(2n-1))}\}$ are solvent. The payments of the banks in $\{b^{\rho(\bar{i})}, \dots, b^{\rho(\bar{i}-(\hat{K}-1))}\}$ are characterized by

$$q_1^{\rho(\bar{i}-k)} = f\left(q_1^{\rho(\bar{i}-(k-1))}\right) \text{ for each } k \in \{1, \dots, \hat{K}-1\}, \quad (32)$$

which is an increasing sequence (by condition (13)). Then, either $q_1^{\rho(\bar{i}-(2n-2))} < x[\bar{y}_0]$ and we are back to subcase 1 (i.e. $\hat{K} = 2n - 1$), or there exists a unique $\hat{K} \in [K, 2n - 1]$ such that

$$q_1^{\rho(\bar{i}-(\hat{K}-2))} < x[\bar{y}_0] \leq q_1^{\rho(\bar{i}-(\hat{K}-1))}. \quad (33)$$

In the latter case, the banks in $\{b^{\rho(\bar{i}-K)}, \dots, b^{\rho(\bar{i}-(\hat{K}-1))}\} \subset B^{uncertain}(\rho)$ are insolvent (since they receive less than $x[\bar{y}_0]$ from their forward neighbor) but the bank $b^{\rho(\bar{i}-\hat{K})}$

is solvent since it receives at least $x[\bar{y}_0]$ from its forward neighbor. The banks in $\{b^{\rho(\bar{i}-(\hat{K}+1))}, \dots, b^{\rho(\bar{i}-(2n-2))}\}$ are also solvent since they receive $l_1 \geq x[\bar{y}_0]$ from their forward neighbors. The informed bank $b^{\rho(\bar{i}+1)}$ is also solvent as in subcase 1. Finally, this analysis implies that the equilibrium is distance based and monotonic, completing the characterization of the date 1 equilibrium in this case.

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