Due Date: Monday, November 14, 2005

1. Consider a two period model. In the first period all agents are identical, supply one unit of labor that produces one unit of consumption good. In the second period with probability π an agent looses any ability to work. Assume that the law of large numbers holds so that in the second period there will be exactly π agents who are not able to work. It is not possible to identify who is able to work in the second period. The agent's utility is $u(c) + v(y)$ if he is able to work and $u(c)$ if he is not able, with $u'(c)$ and $-u''(c) > 0$. The discount factor is β. There is a technology to transfer resources between periods 1 and 2 with the rate of return r. Assume that $r = 1/\beta$.

Let $\{(c^*_1, y^*_1), (c^*_2, y^*_2), (c^*_d, 0), k^*\}$ be the optimal allocation, i.e. the allocation the solves the social planner's problem, for the first period agents, second period agents who are able to work, second period agents who are not able to work, and savings respectively. Assume that $c^*_2 > c^*_1 > c^*_d$. Please answer the following questions, finding where appropriate explicit expressions in terms of the optimal allocations.

(a) Write down the social planner's problem in this economy and show directly, using first order conditions, that it is optimal for the planner to discourage savings. This should be straightforward since there is only one IC in this economy. What are the optimal labor taxes?

(b) Suppose the planner is able to use only linear taxes on savings, and that she tried to implement the optimal allocations using a uniform linear tax on savings in the second period on both able and disabled agents. Show that any such tax would either give the wrong saving incentives for the truth telling type, or not be able to prevent a double-deviation.

(c) Now assume that the planner attempts to implement the optimum with two linear taxes on savings, one on the agent who claims to be disabled, and the other on the agent who claims to be able in the second period. What is the required tax on the disabled person in the second period to prevent the double-deviation? (Hint: Find a tax that satisfies the property that if the agent cheats and double-deviates he still gets (c^*_1, y^*_1) in the first period and $(c^*_d, 0)$ in the second.)

(d) If the planner imposes the savings tax on those who are found to be disabled in part (c), what is the optimal linear savings tax on the able agent? Show that such a tax is negative (i.e. it is a savings subsidy), while the tax for a disabled person is positive.

(e) Suppose that the planner used linear taxes in parts (c) and (d). Find the remaining taxes which are needed to implement the optimal allocations. Show that the total amount of revenues collected by savings taxes (defined as $\pi\tau_d rk^* + (1 - \pi)\tau_2 rk^*$) is zero. Note that this result says that the savings distortion is implemented without collecting any net revenues from taxing saving. Why does this tax system discourage savings?

2. Consider a representative infinitely lived agent model where government can use only linear taxes on consumption, labor and capital $\{\tau_{ct}, \tau_{lt}, \tau_{kt}\}$.

(a) Show without loss of generality that one of these taxes can be normalized to 0 in all periods.

(b) Consider an equilibrium in which taxes on capital and consumption are positive and constant in all periods. Using the result in part (a), show that an equivalent tax system with capital taxes normalized to zero would necessarily imply that consumption taxes diverge to infinity.

3. Consider an individual who lives for two periods and has an additively separable utility function of the form

$$V(C_1, C_2) \quad = \quad \log C_1 + [1/(1+\delta)] \log C_2.$$

Assume that the individual receives an endowment of W_1 at the beginning of the first period, and then divides this endowment between consumption in the two periods. The pretax rate of return is r. The government has just announced that, for the first time, it will impose a capital tax at rate τ on capital income received in period two. The proceeds of this tax will be paid as a lump-sum transfer to the next generation. Individuals alive today do not care about the next generation.

(a) Find the lifetime indirect utility function, $V(W_1, r(1-\tau))$, for an individual who is just beginning life. Using this expression, evaluate the change in initial endowment (W_1) that would be required to make the individual as well off with the capital tax as without it.

(b) What is $dC_1/d\tau$? What conclusions about the welfare cost of capital income taxation can you draw from this finding?

4. Now consider another individual with the same preferences as the person analyzed in problem 3:

$$V(C_1, C_2) \quad = \quad \log C_1 + [1/(1+\delta)] \log C_2.$$

This individual faces a wage income tax, so that the interest rate at which she can borrow and lend is simply r. Assume that her wage income in periods 1 and 2 is fixed at Y_1 and Y_2, and that her labor income tax rate in period 1 is τ_1 while that in period 2 is τ_2. Assume that $\tau_1 > \tau_2$, and that both taxes are linear. Further assume that the individual has access to a "tax avoidance technology" that permits wage income to be shifted from period 1 to period 2. If the individual chooses to shift A dollars from period 1 to period 2, where A is between 0 and Y_1, her taxable income in period 1 will be $Y_1 - A$ and that in period 2 will be $Y_2 + A$. Using the tax avoidance technology is costly; the cost of shifting A dollars is $\beta(A)$. This cost can be viewed as the legal and administrative fees associated with tax avoidance, and it must be paid in period 1.

(a) Find the lifetime budget constraint for this individual, recognizing both the impact of tax avoidance on income net of taxes, and the cost of tax avoidance.

(b) Now obtain first order conditions for the optimal choice of A. Does the optimal level of A depend on the utility function? Explain why or why not.

(c) Consider the case in which $\beta(A) = \gamma A^2$, and assume that $r = 0$. Obtain a formula for A as a function of the tax rates in the two periods, and compute the elasticity of tax avoidance (A) with respect to $(1-\tau_1)$. Briefly describe the implications for the impact of tax changes on revenue.

5. The government in Cloneland faces a long-standing constitutional constraint against levying lump-sum taxes. This is unfortunate, because everyone in Cloneland is identical. The government must raise revenue of R per person to pay tribute to a foreign colonial power that conquered Cloneland many centuries ago. The citizens of Cloneland consume two goods (x_1 and x_2) and they supply labor (L) to earn enough to cover their consumption purchases. The Clones live for only one period, and they all have a wage of unity (so each unit of labor supply earns income of one). The producer prices of the two consumption goods are also equal to unity. The government relies on commodity taxes on goods 1 and 2 at rates τ_1 and τ_2 to raise revenue. The Clones have identical utility functions given by:

$$U = (\alpha_1/(1-1/\varepsilon))*x_1^{1-1/\varepsilon} + (\alpha_2/(1-1/\eta))*x_2^{1-1/\eta} - L.$$

(a) Find the demands for the two consumption goods, and use these demands to write out the government's budget constraint. Then use these demands to obtain the indirect utility function for a representative Clone as a function of the two tax rates.

(b) Determine the relative values of the tax rates on the two goods. This requires finding an expression for τ_1/τ_2 or, what may be easier, $\{\tau_1/(1+\tau_1)\}/\{\tau_2/(1+\tau_2)\}$. You do not need to solve explicitly for each tax rate as a function of preferences and the required revenue level.