Due Date: Wednesday, October 12, 2005

1. You are invited to visit a small island nation and to help in analyzing a proposed tax reform. The economy consists of two types of individuals, with each type accounting for half of the population. Jello is the numeraire consumption good in the economy. The first type of individual has a wage of 1 jello packet per unit of labor supplied, and the second has a wage of 2 jello packets per unit of labor. Individuals of each type have an endowment of one unit of labor. Each individual also has a lump sum income of one packet of jello.

Individuals of both types have preferences over three goods: leisure (1-L), jello (J), and coconuts (C). These preferences are described by a utility function:

$$U = \log J + \log C + \log (1-L).$$

There is a domestic production technology that turns one package of jello into one coconut. The process can be reversed.

The current tax system is as follows. Taxable income, which is measured in units of jello, is defined as labor earnings plus lump sum income. The marginal tax rate on the first 1.5 jellos of income is 25 percent, while the marginal tax rate is 50 percent on all income above this level.

(a) You begin your analysis of the tax system by trying to determine the optimal labor supply behavior of the two types of individuals in this economy. Find the labor supply, consumption of jello, and consumption of coconuts for individuals with a wage of 1, and for those with a wage of 2.

(b) What is per capita tax revenue from the current tax code?

(c) The domestic coconut lobby has proposed a tax deduction for coconut purchases. If all expenditures on coconuts were excluded from taxable income, describe the new tax schedule. How would per capita revenue change if this policy were adopted? Explain the factors that contribute to any revenue change that you identify.

(d) The coconut lobby has evaluated the potential revenue cost of the proposal in (c) by multiplying outlays on coconuts under the current (no coconut deduction) tax system by estimates of marginal tax rates for consumers of types 1 and 2. (This is known as "static revenue estimation.") Do your calculations offer any insights on the validity or limitations of this approach?

2. Consider a variant of the optimal commodity taxation model discussed in class. There are I consumption goods and leisure. The utility function is given by $u(c_1,\ldots,c_I) - v(l)$, where c_i is consumption of good i and l is labor. Assume that production function is $F(c_1,\ldots,c_I,l) = c_1 + \ldots + c_I - l$, so that all prices are equal to unity in competitive equilibrium. Government needs to finance purchases of goods $g_1,\ldots g_I$.

(a). Suppose government currently uses linear taxes on each consumption good t_i *and* labor t_l. Show that the government could change the tax schedule in such a way that the tax on one of the consumption goods or on labor is equal to zero, and that taxes on the other goods collect the same amount of revenues, while equilibrium allocations remain the same.

(b). Suppose we normalize tax on good 1 to zero. Under what conditions is it optimal to set taxes on the other goods to be equal to zero as well? [Hint: formulas we derived in class for the optimal commodity taxation could be helpful]

(c). Alternatively, suppose that the tax on labor is normalized to zero. In addition assume that the utility function has a quasilinear form $u^1(c_1) +\ldots+ u^I(c_I) - v(l)$. Define the price elasticity of demand for good i as $\varepsilon_i = -(\partial c_i/\partial p_i)p_i/c_i$. Show that goods with relatively low ε_i should be taxed relatively more heavily. [Hint: In class we showed that taxes are proportional to $H_i = -U_{ii}c_i/U_i$ when utility is separable. Show that in the quasilinear case, $H_i = 1/\varepsilon_i$]. How is this result related to the intuition one could get about optimal taxation from partial equilibrium analysis?

3. Consider an economy with two types of workers and assume that the social planner has a utilitarian social welfare function, W(u)=u. Assume that the government does not need to collect any revenues.

(a) We showed in class that the optimal non-linear labor tax T(y) should impose no distortion for the high type and distort labor decisions of the low type. Suppose the solution to the social planner's (SP) problem is $(c^*_h, c^*_l, y^*_h, y^*_l)$. Give an example of an income tax function T(y) so that high types choose (c^*_h, y^*_h) in equilibrium and low types choose (c^*_l, y^*_l). (Hint: In class we showed that such a function should satisfy $T'(y^*_l) = 1 - v'(y^*_l)/w_l u'(c^*_l)$ and $T'(y^*_h)=0$. You need to define T(y) for all feasible y and verify that it indeed implements the optimum.)

(b) Show that instead of using labor income tax T(y) the government can use a consumption tax T(c) to implement the optimal allocations. Find such a tax.

(c) Suppose there are two consumption goods, c_1 and c_2. A worker with skill w_i can produce any combination of these goods as long as they satisfy $c_1+c_2=w_i l_i$. Prove a version of the Uniform Commodity Taxation theorem, which states that as long as the utility function is additive between consumption and labor, i.e. takes the form $u(c_1, c_2, l) = u(c_1, c_2) + v(l)$, all consumption goods are taxed at the same rate.

(d) Compare your analysis of (c) in this problem to your analysis in (b) of the last problem. What are the implications of this comparison for the optimal commodity taxation?

(e) Suppose the economy lasts for two periods instead of one period. Each agent has a fixed type, which is the same in periods one and two. The utility function is

$$U = u(c_1) - v(l_1) + \beta(u(c_1) - v(l_1))$$

where $0<\beta<1$ is a discount factor. It is not possible to transfer resources between periods. Write down the social planner's problem and show that the optimal taxes in both periods are the same as the optimal tax in static economy.

4. As news of your success in solving the foregoing problems spreads, you are invited to a small tropical island nation to become a tax policy consultant, with particular responsibility for analyzing tax-induced distortions in labor supply. Sensing the opportunity for an interesting vacation, you accept. The republic's finance minister faxes you a copy of the current tax schedule, defined over total daily (labor plus non-labor) income, which is:

$$T(y) = \begin{cases} 0 & \text{if } y < 60 \\ .5*(y - 60) & \text{if } y > 60. \end{cases}$$

After arriving in the republic, you discover it is smaller than you thought. The Complete Population Survey contains only four observations. Undaunted, you proceed with your proposal to estimate a linear hours-of-work model for the republic's population:

$$h = \alpha + \beta y_v + \gamma w$$

where h denotes daily hours, y_v is virtual income, and w is the household's after-tax wage rate. The data you receive are shown below:

Household	Non-labor Income	Hours Worked	Total Pretax Income
1	0	15.15	45.45
2	0	10.00	60.00
3	10	8.25	92.50
4	20	7.25	92.50.

(a) Use these data to estimate (α, β, γ). You should be able to solve for the parameters exactly (i.e., with no error terms in the hours equation) using three data points. (Hint: Remember to check whether each household is on a linear segment of the budget set, or at a "kink point.")

(b) Compute the total amount of revenue currently collected by the tax system, and find the lump sum tax (equal across all households) that would be needed to raise the same amount of revenue. For household 1, find the equivalent variation of shifting to this tax.

5. An economy is made up of individuals who have identical preferences, but different wage rates. Everyone has preferences given by the indirect utility function:

$$v(w,y) = e^{-.04*w}\{y - 12.5*w - 412.50\}$$

where w denotes the per-hour after-tax wage rate, and y denotes nonlabor income. No one in the economy has any non-labor income. Half of the population has an hourly wage rate of 10, while the other half has an hourly wage of 20.

The economy has the following income tax schedule:

$$T(Y) = \begin{cases} .10*Y & Y < 90 \\ 9 + .5(Y-90) & Y >= 90. \end{cases}$$

(a) Use the indirect utility function to find the labor supply function for everyone in this economy. (Note that this is a function of each person's wage rate and nonlabor income. You should find a positive number of hours of work.)

(b) Find the number of hours of labor supply for individual's with a wage of 10, and for those with a wage of 20.

(c) How much revenue does the government collect from the existing income tax system?

(d) Now imagine that in the name of fiscal stimulus, policymakers in this economy propose eliminating the lower bracket of the income tax, so that $T(Y) = 0$ if $Y < 90$. Describe the budget set for individuals with wages of 10, and with wages of 20, after this change.

(e) Does elimination of the lower bracket in part (d) change the virtual income of individuals who were previously earning more than 90? Explain why or why not.