Due Date: Monday, September 26, 2005

1. Assume that a consumer's utility function is given by:

$$u(x_1, x_2, x_3) = \beta_1 \log (x_1 - \alpha_1) + \beta_2 \log (x_2 - \alpha_2) + \beta_3 \log x_3$$

and that the consumer faces consumer prices q_1 and q_2, with the price of good three normalized to unity. The consumer's endowment (y) is measured in units of good three.

(a) Find the indirect utility function and expenditure function corresponding to this set of preferences. (This is the famous Stone-Geary utility function, the basis for the linear expenditure system in demand analysis.)

(b) Use your results from (a) to find an analytic expression for the compensating variation. In the special case of $\alpha_1 = \alpha_2 = .50$ and $\beta_1 = \beta_2 = .40$, find the CV associated with a tax reform that changes (q_1, q_2, y) from (1, 1, 5) to (2.0, 1.5, 5). How would you expect the EV to compare with the CV in this case (you do not need to compute the EV, but simply to note whether you believe it would be larger or smaller, and why).

(c) Assuming that the differences between the pre- and post-reform consumer prices in (b) are due only to tax changes, i.e. that producer prices are fixed, compute the government's revenue from the new tax policy.

2. Consider a world economy consisting of N identical countries, each endowed with one unit of land. The world contains one unit of capital, which is freely mobile between countries. Land, in contrast, is immobile. All countries have identical production technologies given by

$$Y = K^{.25}L^{.75}$$

where K and L denote capital and land, respectively. The output price is normalized to unity.

(a) Find the equilibrium interest rate and total income received by capitalists and landowners when none of the jurisdictions tax either capital or land. Evaluate these quantities for N = 2 and N = 100. (Note that N = 100 means there is more land in the world than N=2!)

(b) Now consider the impact of a tax at rate θ on capital income in country 1. Assume that revenues are used to purchase good Y, and that the government's purchases do not affect the production technology in country 1. The after-tax rate of return to capital invested in country 1 is now $(1-\theta)F_K$. Find new expressions for the after-tax interest rate, total landowner and capitalist income, and government revenue in country 1 as functions of N and θ.

(c) For $\theta = .25$, find the change in total capital and total land income, and the revenue raised in country 1, if N = 2 and N = 100. What happens to the pretax marginal product of capital in the countries without taxes? How do landowners in country 1 fare as a result of the tax? What do these examples suggest about the usefulness of the "small open economy" assumption that world interest rates are fixed, so capital taxes are shifted to land?

3. Many analysts believe that tax rules have an important effect on the market for apartment buildings. Assume that there is a homogeneous stock of apartment units, with a price per rental unit of P_t and a stock of such units H_t. Let I denote gross investment in new apartment units. Gross investment as a share of the existing stock of units is an increasing function of the price of apartment units:

$$I_t / H_t = \beta_0 + \beta_1 * P_t \ .$$

The rental value of an apartment unit is a declining function of the stock of units: R = R(H), where $R(H) = H^{-1}$. This corresponds to a unit elastic demand for rental apartment units. Assume that landlords use debt to finance their apartment units, that interest on such debt is fully deductible against income taxes, that the (time invariant) interest rate is r, and that the marginal income tax rate facing landlords is τ. In addition to the after-tax cost of borrowing, the landlord faces depreciation charges, as apartments depreciate at a constant rate δ per year, and receives capital gains or losses if the price of apartment units changes. The net change in the stock of apartment units equals gross investment I_t minus depreciation on the existing stock, $\delta * H_t$.

(a) Find the equilibrium condition that ensures that landlords will hold the entire stock of apartment units at all points in time. Express this in discrete time, so that the equation relates current (and possibly past or future) values of the real price of apartment units and the stock of such units.

(b) Using the equilibrium condition in (a) along with the equation of motion for the number of rental housing units, $H_{t+1} = I_t + (1-\delta) * H_t$, find the steady state value for the real price of apartment units and the stock of such units. You should express P* and H* as functions of r, δ, τ, and the parameters of the gross investment function.

(c) Imagine that tax reform eliminated the tax deduction for the mortgage interest associated with owning an apartment building, but that pretax interest rates were not affected by this reform so the after-tax cost of borrowing rose from $(1-\tau)r$ to r. Assume that landlords continue to borrow to finance apartment units. How would this reform affect the steady state stock of rental housing units? How would it change the steady state price of rental units?

(d) If housing did not depreciation ($\delta = 0$), how would the reform in (c) affect the price of rental units?

(e) (OPTIONAL) Now consider price dynamics for the case of $r = \delta = .04$, $\beta_0 = .01$, and $\beta_1 = .03$. Solve the equation of motion for the number of rental units and the equilibrium condition you found in (a) so that you have a two equation system relating $\{P_{t+1}, H_{t+1}\}$ to $\{P_t, H_t\}$. Starting from the steady state value of H* in (b), write a simple computer program that will iteratively solve for $\{P_{t+j}, H_{t+j}\}$ if given $\{P_{t+j-1}, H_{t+j-1}\}$. Use this simple algorithm to search for the saddlepoint stable path that will lead from the initial steady state when interest is deductible, the equilibrium point {P*, H*} from (b) above, to the new steady state when interest is no longer deductible.