Due Date: Wednesday, December 1, 2004

1. Consider an economy where investors demand a required return of 15 percent per year, capital depreciates at 10 percent per year, and output (y) is related to capital (k) according to the production function $y = k^{0.5}$. Show that the optimal capital stock when capital investment can be expensed under the corporate income tax is $k^* = 4$.

2. Consider an economy in which firms produce output using two capital inputs, equipment (E) and structures (S), according to the production technology

$$Y = E^{.25}S^{.25}.$$

Assume that the price of output is fixed at unity and that both equipment and structures are in infinitely elastic supply with a price of unity. Investors demand a required return of 10 percent on all capital investments, and both equipment and structures depreciate at a rate of 15 percent per year. Firms are 100 percent equity financed. In the initial setting, the tax rate on corporate profits is 50 percent and equipment investment can be expensed, while investments in structures are eligible for depreciation allowances equal to true economic depreciation.

(a) Find the numerical values that the pretax marginal products of equipment and structures must satisfy.

(b) Calculate the firm's output if both equipment and structures could be expensed, and compare this with the output in the case when equipment can be expensed while structures are depreciated using true economic depreciation.

(c) Assuming that firms are in a steady state in which their gross investment is precisely equal to the depreciation on their existing capital stock, compare the revenue that would be collected by the corporate income tax when both types of investment can be expensed, and when only equipment can be expensed while structures receive true economic depreciation. Recall that in computing tax revenue, you should calculate τ*[F(E,S) - Depreciation Deductions].

3. Consider a two period model. In the first period all agents are identical, supply one unit of labor that produces one unit of consumption good. In the second period with probability π an agent looses any ability to work. Assume that the law of large numbers holds so that in the second period there will be exactly π agents who are not able to work. It is not possible to identify who is able to work in the second period. The agent's utility is $u(c) + v(y)$ if he is able to work and $u(c)$ if he is not able, with $u'(c)$ and $-u''(c) > 0$. The discount factor is β. There is a technology to transfer resources between periods 1 and 2 with the rate of return r. Assume that $r = 1/\beta$.

Let $\{(c^*_1, y^*_1), (c^*_2, y^*_2), (c^*_d, 0), k^*\}$ be the optimal allocation, i.e. the allocation that solves the social planner's problem, for the first period agents, second period agents who are able to work, second period agents who are not able to work, and savings respectively. Assume that $c^*_2 > c^*_1 > c^*_d$. Please answer the following questions, finding where appropriate explicit expressions in terms of the optimal allocations.

(a) Write down the social planner's problem in this economy and show directly, using first order conditions, that it is optimal for the planner to discourage savings. This should be straightforward since there is only one IC in this economy. What are the optimal labor taxes?

(b) Suppose the planner is able to use only linear taxes on savings, and that she tried to implement the optimal allocations using a uniform linear tax on savings in the second period on both able and disabled agents. Show that any such tax would either give the wrong saving incentives for the truth telling type, or not be able to prevent a double-deviation.

(c) Now assume that the planner attempts to implement the optimum with two linear taxes on savings, one on the agent who claims to be disabled, and the other on the agent who claims to be able in the second period. What is the required tax on the disabled person in the second period to prevent the double-deviation? (Hint: Find a tax that satisfies the property that if the agent cheats and double-deviates he still gets (c^*_1, y^*_1) in the first period and $(c^*_d, 0)$ in the second.)

(d) If the planner imposes the savings tax on those who are found to be disabled in part (c), what is the optimal linear savings tax on the able agent? Show that such a tax is negative (i.e. it is a savings subsidy), while the tax for a disabled person is positive.

(e) Suppose that the planner used linear taxes in parts (c) and (d). Find the remaining taxes which are needed to implement the optimal allocations. Show that the total amount of revenues collected by savings taxes (defined as $\pi\tau_d r k^* + (1 - \pi)\tau_2 r k^*$) is zero. Note that this result says that the savings distortion is implemented without collecting any net revenues from taxing saving. Why does this tax system discourage savings?

4. Consider a world economy consisting of N identical countries, each endowed with one unit of land. The world contains one unit of capital, which is freely mobile between countries. Land, in contrast, is immobile. All countries have identical production technologies given by

$$Y = K^{.25}L^{.75}$$

where K and L denote capital and land, respectively. The output price is normalized to unity.

(a) Find the equilibrium interest rate and total income received by capitalists and landowners when none of the jurisdictions tax either capital or land. Evaluate these quantities for N = 2 and N = 100. (Note that N = 100 means there is more land in the world than N=2!)

(b) Now consider the impact of a tax at rate θ on capital income in country 1. Assume that revenues are used to purchase good Y, and that the government's purchases do not affect the production technology in country 1. The after-tax rate of return to capital invested in country 1 is now $(1-\theta)F_K$. Find new expressions for the after-tax interest rate, total landowner and capitalist income, and government revenue in country 1 as functions of N and θ.

(c) For $\theta = .25$, find the change in total capital and total land income, and the revenue raised in country 1, if N = 2 and N = 100. What happens to the pretax marginal product of capital in the countries without taxes? How do landowners in country 1 fare as a result of the tax? What do these examples suggest about the usefulness of the "small open economy" assumption that world interest rates are fixed, so capital taxes are shifted to land?