

UNCERTAIN OUTCOMES AND CLIMATE CHANGE POLICY*

by

Robert S. Pindyck
Massachusetts Institute of Technology
Cambridge, MA 02142

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Abstract: Focusing on tail effects, I incorporate distributions for temperature change and its economic impact in an analysis of climate change policy. I estimate the fraction of consumption $w^*(\tau)$ that society would be willing to sacrifice to ensure that any increase in temperature at a future point is limited to τ . Using information on the distributions for temperature change and economic impact from studies assembled by the IPCC and from “integrated assessment models” (IAMs), I fit displaced gamma distributions for these variables. Unlike existing IAMs, I model economic impact as a relationship between temperature change and the growth rate of GDP as opposed to its level, so that warming has a permanent impact on future GDP. The fitted distributions for temperature change and economic impact generally yield values of $w^*(\tau)$ below 2%, even for small values of τ , unless one assumes extreme parameter values and/or substantial shifts in the temperature distribution. These results are consistent with moderate abatement policies.

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1 Introduction.

Economic analyses of climate change policies often focus on a set of “likely” scenarios — those within a roughly 66 to 90 percent confidence interval — for emissions, increases in temperature, economic impacts, and abatement costs. It is hard to justify the immediate adoption of a stringent abatement policy given these scenarios and consensus estimates of discount rates and other relevant parameters.¹ I ask whether a stringent policy might be justified by a cost-benefit analysis that accounts for a full distribution of possible outcomes.

Recent climate science and economic impact studies provide information about less likely scenarios, and allow one to at least roughly estimate the distributions for temperature change and its economic impact. I show how these distributions can be incorporated in and affect conclusions from analyses of climate change policy. As a framework for policy analysis, I estimate a simple measure of “willingness to pay” (WTP): the fraction of consumption $w^*(\tau)$ that society would be willing to sacrifice, now and throughout the future, to ensure that any increase in temperature at a specific horizon H , ΔT_H , is limited to τ . Whether the reduction in consumption corresponding to a particular $w^*(\tau)$ is *sufficient* to limit warming to τ is a separate question which I do not address. Thus I avoid having to make projections of GHG emissions and atmospheric concentrations, or estimate abatement costs. Instead I focus directly on uncertainties over temperature change and its economic impact.²

My analysis is based on the current “state of knowledge” regarding global warming and its impact. In particular, I use information on the distributions for temperature change from scientific studies assembled by the IPCC (2007) and information about economic impacts from recent “integrated assessment models” (IAMs) to fit displaced gamma distributions for these variables. But unlike existing IAMs, I model economic impact as a relationship between temperature change and the *growth rate* of GDP as opposed to the level of GDP.

¹An exception is the Stern Review (2007), but as Nordhaus (2007), Weitzman (2007), Mendelsohn (2008) and others point out, that study makes assumptions about temperature change, economic impact, abatement costs, and discount rates that are well outside the consensus range.

²By “economic impact” I mean to include any adverse impacts resulting from global warming, such as social, medical, or direct economic impacts.

This distinction is justified on theoretical and empirical grounds, and implies that warming can have a permanent impact on future GDP. I then examine whether “reasonable” values for the remaining parameters (e.g., the starting growth rate and the index of risk aversion) can yield values of $w^*(\tau)$ above 2 or 3% for small values of τ , which might support stringent abatement. Also, by transforming the displaced gamma distributions, I show how $w^*(\tau)$ depends on the mean, variance, and skewness of each distribution, which provides additional insight into how uncertainty drives WTP.

To explore the case for stringent abatement, I use a counterfactual — and pessimistic — scenario for temperature change: Under “business as usual” (BAU), the atmospheric GHG concentration immediately increases to twice its pre-industrial level, which leads to an (uncertain) increase in temperature at the horizon H , and then (from feedback effects or further emissions) a gradual further doubling of that temperature increase.

This paper builds on recent work by Weitzman (2009a), but takes a very different approach. Suppose there is some underlying probability distribution for temperature change, ΔT , but its variance is unknown and is estimated through ongoing Bayesian learning. Weitzman shows that this “structural uncertainty” implies that the posterior-predictive distribution of ΔT is “fat-tailed,” i.e., approaches zero at a less than exponential rate (and thus has no moment generating function). If welfare is given by a power utility function, this means that the expected loss of future welfare from warming is infinite, so that society should be willing to sacrifice *all* current consumption to avoid future warming.³

Weitzman provides insight into the nature of the uncertainty underlying climate change policy, but his results do not readily translate into a policy prescription, e.g., what percentage of consumption society should sacrifice to avoid warming. What his results *do* tell us is that the right-hand tail of the distribution for ΔT may matter most for policy, and we know very little about that tail. In other words, because of its focus on the middle of the distribution of outcomes, traditional cost-benefit analysis may be misleading.

³In another paper, Weitzman (2009b) presents an alternative argument, based on the underlying mechanism of GHG accumulation and its effect on temperature, for why the distribution of ΔT should be fat-tailed, but this has the same disturbing welfare implications. For a related discussion of inherent uncertainty over climate sensitivity, and a model that implies a fat-tailed distribution for ΔT , see Roe and Baker (2007).

I utilize a (thin-tailed) three-parameter displaced gamma distribution for temperature change, which I calibrate using estimates of its mean and confidence intervals inferred from the studies surveyed by the IPCC. Besides its simplicity and reasonable fit to the IPCC studies, this approach has two advantages. First, a thin-tailed distribution avoids infinite welfare losses (or the need to arbitrarily bound the utility function to avoid infinite losses). Second, the skewness or variance of the distribution can be altered while holding the other moments fixed, providing additional insight into tail effects.

I specify an economic impact function that relates temperature change to the growth rate of GDP and consumption, and calibrate the relationship using damage functions from several IAMs. Although these damage functions are based on levels of GDP, I can calibrate a growth rate function by matching estimates of GDP/temperature change pairs at a specific horizon. I then use the distribution of GDP level reductions at that horizon to fit a displaced gamma distribution for the growth rate impact.

After fitting gamma distributions to temperature change and growth rate impact, I calculate WTP based on expected discounted utility, using a constant relative risk aversion (CRRA) utility function. In addition to the initial growth rate and index of risk aversion, WTP is affected by the rate of time preference (the rate at which future utility is discounted). I set this rate to zero, the “reasonable” (if controversial) value that gives the highest WTP.⁴

My estimates of $w^*(\tau)$ are generally below 2%, even for τ around 2 or 3°C. This is because there is limited weight in the tails of the calibrated distributions for ΔT and growth rate impact. Larger estimates of WTP result for particular combinations of parameter values (e.g., an index of risk aversion close to 1 and a low initial GDP growth rate), or if I assume an accelerated rate of warming (i.e., the distribution for ΔT applies to a shorter horizon).

⁴Newbold and Daigneault (2008) also studied implications of uncertainty for climate change policy. They combined a distribution for ΔT with CRRA utility and functions that translate ΔT into lost consumption to estimate WTP. They assume there is a “true” value for ΔT and focus on how distributions from different studies could be combined to obtain a (Bayesian) posterior distribution. They solve for the parameters of a distribution derived by Roe and Baker (2007) for each of 21 studies that estimated 5th and 95th percentiles, and combined the resulting distributions in two ways: (1) averaging them, which (“pessimistically”) assumes the studies used the same data but different models, and yields a relatively diffuse posterior distribution; and (2) multiplying them, which (“optimistically”) assumes the studies used the same model but independent datasets, and yields a relatively tight posterior distribution.

But overall, given the current “state of knowledge” of warming and its impact, my results are consistent with moderate abatement. Of course the “state of knowledge” is evolving and new studies might lead to changes in the distributions. The framework developed here could then be used to evaluate the policy implications of such changes.

This paper ignores the implications of the opposing irreversibilities inherent in climate change policy and the value of waiting for more information. Immediate action reduces the largely irreversible build-up of GHGs in the atmosphere, but waiting avoids an irreversible investment in abatement capital that might turn out to be at least partly unnecessary, and the net effect of these irreversibilities is unclear. I focus instead on the nature of the uncertainty and its application to a relatively simple cost-benefit analysis.⁵

The next section explains in more detail the methodology used in this paper and its relationship to other studies of climate change policy. Section 3 discusses the probability distribution for temperature change and how it can be transformed to estimate mean, variance and skewness effects. Section 4 discusses the economic impact function and the corresponding uncertainty. Section 5 shows estimates of willingness to pay and its dependence on free parameters, and Section 6 concludes.

2 Background and Methodology.

Most economic analyses of climate change policy have five elements: (1) Projections of future emissions of a CO₂ equivalent (CO₂e) composite (or individual GHGs) under a “business as usual” (BAU) and one or more abatement scenarios, and resulting future atmospheric CO₂e concentrations. (2) Projections of the average or regional temperature changes likely to result from higher CO₂e concentrations. (3) Projections of lost GDP and consumption resulting from higher temperatures. (This is probably the most speculative element because

⁵A number of studies have examined the policy implications of this interaction of uncertainty and irreversibility, but with mixed results, showing that policy adoption might be delayed or accelerated. See, for example, Kolstad (1999b), Gollier, Jullien and Treich (2000), and Fisher and Narain (2003), who use two-period models for tractability; and include Kolstad (1996a), Pindyck (2000, 2002) and Newell and Pizer (2003), who use multi-period or continuous-time models. For a discussion of these and other studies of the interaction of uncertainty and irreversibility, see Pindyck (2007).

of uncertainty over adaptation to climate change, e.g., through shifts in agriculture, migration, etc.) (4) Estimates of the cost of abating GHG emissions by various amounts. (5) Assumptions about social utility and the rate of time preference, so that lost consumption from abatement can be weighed against future gains in consumption from reduced warming. This is essentially the approach of Nordhaus (1994, 2008), Stern (2007), and others who evaluate abatement policies using integrated assessment models (IAMs) that project emissions, CO₂e concentrations, temperature change, economic impact, and costs of abatement.

Each of these five elements of an IAM-based analysis is subject to considerable uncertainty. However, by estimating WTP instead of evaluating specific policies, I avoid having to deal with abatement costs and projections of GHG emissions. Instead, I focus on uncertainty over temperature change and its economic impact as follows.

2.1 Temperature Change.

According to the most recent IPCC report (2007), growing GHG emissions would likely lead to a doubling of the atmospheric CO₂e concentration relative to the pre-industrial level by the end of this century. That, in turn, would cause an increase in global mean temperature that would “most likely” range between 1.0°C to 4.5°C, with an expected value of 2.5°C to 3.0°C. The IPCC report indicates that this range, derived from a “summary” of the results of 22 scientific studies the IPCC surveyed, represents a roughly 66- to 90-percent confidence interval, i.e., there is a 5 to 17-percent probability of a temperature increase above 4.5°C.⁶

The 22 studies themselves also provide rough estimates of increases in temperature at the outer tail of the distribution. In summarizing them, the IPCC translated the implied outcome distributions into a standardized form that allows comparability across the studies, and created graphs showing multiple outcome distributions implied by groups of studies. As Weitzman (2008) has argued, those distributions suggest that there is a 5% probability that a doubling of the CO₂e concentration relative to the pre-industrial level would lead

⁶The atmospheric CO₂e concentration was about 300 ppm in 1900, and is now about 370 ppm. The IPCC (2007) projects an increase to 550 to 600 ppm by 2100. The text of the IPCC report is vague as to whether the 1.0°C to 4.5°C “most likely” range for ΔT in 2100 represents a 66% or a 90% confidence interval.

to a global mean temperature increase of 7°C or more, and a 1% probability that it would lead to a temperature increase of 10°C or more. I fit a three-parameter displaced gamma distribution for ΔT to these 5% and 1% points and to a mean temperature change of 3.0°C. This distribution conforms with the distributions summarized by the IPCC, and can be used to study “tail effects” by calculating the impact on WTP of changes in the distribution’s variance or skewness (holding the other moments fixed).

I assume that the fitted gamma distribution for ΔT applies to a 100-year horizon H and that $\Delta T_t \rightarrow 2\Delta T_H$ as t gets large. This implies that ΔT_t follows the trajectory:⁷

$$\Delta T_t = 2\Delta T_H [1 - (1/2)^{t/H}] , \tag{1}$$

Thus if $\Delta T_H = 5^\circ\text{C}$, ΔT_t reaches 2.93°C after 50 years, 5°C after 100 years, 7.5°C after 200 years, and then gradually approaches 10°C.

2.2 Economic Impact.

Most economic studies of climate change relate ΔT to GDP through a “loss function” $L(\Delta T)$, with $L(0) = 1$ and $L' < 0$, so that GDP at some horizon H is $L(\Delta T_H)\text{GDP}_H$, where GDP_H is but-for GDP in the absence of warming. These studies typically use an inverse-quadratic or exponential-quadratic function.⁸ This implies that if temperatures rise but later fall, GDP could return to its but-for path with no permanent loss.

There are reasons to expect warming to affect the growth rate of GDP as opposed to the level. First, some effects of warming are likely to be permanent: for example, destruction of ecosystems from erosion and flooding, extinction of species, and deaths from health effects and weather extremes. Second, resources needed to counter the impact of higher temperatures would reduce those available for R&D and capital investment, reducing growth.

⁷This allows for possible feedback effects and/or further emissions. As summarized in Weitzman (2009b), the simplest dynamic model relating ΔT_t to the GHG concentration G_t is the differential equation

$$d\Delta T/dt = m_1[\ln(G_t/G_0)/\ln 2 - m_2\Delta T_t] .$$

Assuming G_t initially doubles to $2G_0$, $\Delta T_t = \Delta T_H$ at $t = H$, and $\Delta T_t \rightarrow 2\Delta T_H$ as $t \rightarrow \infty$, implies eqn. (1).

⁸The inverse-quadratic loss function used in the current version of the Nordhaus (2008) DICE model is $L = 1/[1 + \pi_1\Delta T + \pi_2(\Delta T)^2]$. Weitzman (2008) introduced the exponential loss function $L(\Delta T) = \exp[-\beta(\Delta T)^2]$, which, as he points out, allows for greater losses when ΔT is large.

Adaptation to rising temperatures is equivalent to the cost of increasingly strict emission standards, which, as Stokey (1998) has shown with an endogenous growth model, reduces the rate of return on capital and lowers the growth rate.⁹

Finally, there is empirical support for a growth rate effect. Using historical data on temperatures and precipitation over the past 50 years for a panel of 136 countries, Dell, Jones, and Olken (2008) have shown that higher temperatures reduce GDP growth rates but not levels. The impact they estimate is large — a decrease of 1.1 percentage points of growth for each 1°C rise in temperature — but significant only for poorer countries.¹⁰

I assume that in the absence of warming, real GDP and consumption would grow at a constant rate g_0 , but warming will reduce this rate:

$$g_t = g_0 - \gamma \Delta T_t \quad (2)$$

This simple linear relation was estimated by Dell, Jones, and Olken (2008), and can be viewed as at least a first approximation to a more complex loss function.

Uncertainty is introduced into eqn. (2) through the parameter γ . Using information from a number of IAMs, I obtain a distribution for β in the exponential loss function:

$$L(\Delta T) = e^{-\beta(\Delta T)^2} \quad , \quad (3)$$

which applies to the *level* of GDP. I translate this into a distribution for γ using the trajectory for GDP and consumption implied by eqn. (2) for a temperature change-impact combination projected to occur at horizon H . From eqns. (1) and (2), the growth rate is $g_t = g_0 - 2\gamma \Delta T_H [1 - (1/2)^{t/H}]$. Normalizing initial consumption at 1, this implies:

$$C_t = e^{\int_0^t g(s) ds} = \exp \left\{ -\frac{2\gamma H \Delta T_H}{\ln(1/2)} + (g_0 - 2\gamma \Delta T_H)t + \frac{2\gamma H \Delta T_H}{\ln(1/2)} (1/2)^{t/H} \right\} \quad . \quad (4)$$

⁹Suppose total capital $K = K_p + K_a(T)$, with $K'_a(T) > 0$, where K_p is directly productive capital and $K_a(T)$ is capital needed for adaptation to the temperature T (e.g., stronger retaining walls and pumps to counter flooding, new infrastructure and housing to support migration, more air conditioning and insulation, etc.). If all capital depreciates at rate δ_K , $\dot{K}_p = \dot{K} - \dot{K}_a = I - \delta_K K - K'_a(T)\dot{T}$, so that the rate of growth of K_p is reduced. See Brock and Taylor (2004) for a related analysis.

¹⁰“Poor” means below median PPP-adjusted per-capita GDP. Using World Bank data for 209 countries, “poor” accounts for 26.9% of 2006 world GDP, which implies a roughly 0.3 percentage point reduction in world GDP growth for each 1°C rise in temperature. In a later paper (2009), they estimate a model with adaptation effects, so the long-run impact of warming is smaller than the short-run impact. They find a long-run decrease of 0.51 percentage points of poor-country growth for each 1°C rise in temperature.

Thus γ is obtained from β by equating the expressions for C_H implied by eqns. (3) and (4):

$$\exp \left\{ -\frac{2\gamma H \Delta T_H}{\ln(1/2)} + (g_0 - 2\gamma \Delta T_H)H + \frac{\gamma H \Delta T_H}{\ln(1/2)} \right\} = \exp\{g_0 H - \beta(\Delta T_H)^2\} , \quad (5)$$

so that $\gamma = 1.79\beta\Delta T_H/H$.

The IPCC does not provide standardized distributions for lost GDP corresponding to any particular ΔT , but it does survey the results of several IAMS. As discussed in Section 4, I use the information from the IPCC along with other studies to infer means and confidence intervals for β and thus γ . As with ΔT , I fit a displaced gamma distribution to the parameter γ , which I use to study implications of impact uncertainty on WTP.

2.3 Willingness to Pay.

Given the distributions for ΔT and γ , I posit a CRRA social utility function:

$$U(C_t) = C_t^{1-\eta}/(1-\eta) , \quad (6)$$

where η is the index of relative risk aversion (and $1/\eta$ is the elasticity of intertemporal substitution). I calculate the fraction of consumption — now and throughout the future — society would sacrifice to ensure that any increase in temperature at a specific horizon H is limited to an amount τ . That fraction, $w^*(\tau)$, is the measure of willingness to pay.¹¹

An issue in debates over climate change policy is the social discount rate (SDR) on consumption. The Stern Review (2007) used a rate just over 1 percent; critiques by Nordhaus (2007), Weitzman (2007) and others argue for a rate closer to the private return on investment (PRI), around 5 to 6 percent. As Stern (2008) makes clear, the SDR could differ from the PRI, in part because a social investment can affect the consumption trajectory. In my model the consumption discount rate is endogenous; in the Ramsey growth context,

$$R_t = \delta + \eta g_t = \delta + \eta g_0 - 2\eta\gamma\Delta T_H[1 - (1/2)^{t/H}] , \quad (7)$$

¹¹The use of WTP as a welfare measure goes back at least to Debreu (1954), was used by Lucas (1987) to estimate the welfare cost of business cycles, and was used in the context of climate change (with $\tau = 0$) by Heal and Kriström (2002) and Weitzman (2008).

where δ is the rate of time preference. Thus R_t falls as ΔT increases.¹² The “correct” value of δ is itself a subject of debate; I will generally set $\delta = 0$ in order to determine whether any combination of “reasonable” parameter values can yield a high WTP.

If ΔT_H and γ were known, social welfare would be given by:

$$W = \int_0^\infty U(C_t) e^{-\delta t} dt = \frac{1}{1-\eta} \int_0^\infty e^{\omega - \rho t - \omega(1/2)^{t/H}} dt , \quad (8)$$

where

$$\rho = (\eta - 1)(g_0 - 2\gamma\Delta T_H) + \delta , \quad (9)$$

$$\omega = 2(\eta - 1)\gamma H \Delta T_H / \ln(1/2) . \quad (10)$$

Suppose society sacrifices a fraction $w(\tau)$ of present and future consumption to ensure that $\Delta T_H \leq \tau$. With uncertainty, social welfare at $t = 0$ would be:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} \mathcal{E}_{0,\tau} \int_0^\infty e^{\tilde{\omega} - \tilde{\rho} t - \tilde{\omega}(1/2)^{t/H}} dt , \quad (11)$$

where $\mathcal{E}_{0,\tau}$ denotes the expectation at $t = 0$ over the distributions of ΔT_H and γ conditional on $\Delta T_H \leq \tau$. (I use tildes to denote that ρ , and ω are functions of two random variables.)

If no action is taken to limit warming, social welfare would be:

$$W_2 = \frac{1}{1-\eta} \mathcal{E}_0 \int_0^\infty e^{\tilde{\omega} - \tilde{\rho} t - \tilde{\omega}(1/2)^{t/H}} dt , \quad (12)$$

where \mathcal{E}_0 again denotes the expectation over ΔT_H and γ , but now with ΔT_H unconstrained. Willingness to pay to ensure that $\Delta T_H \leq \tau$ is the value $w^*(\tau)$ that equates $W_1(\tau)$ and W_2 .¹³

2.4 Policy Implications.

The case for any abatement policy will depend as much on the cost of that policy as it does on the benefits. I do not estimate abatement costs — I only estimate WTP as a function

¹²If $2\eta\gamma\Delta T_H > \delta + \eta g_0$, R_t becomes negative as ΔT grows. This is entirely consistent with the Ramsey growth model, as pointed out by Dasgupta et al (1999). They provide a simple example in which climate change results in a 2% annual decline in global consumption, and thus a negative consumption discount rate. Of course other models also imply a declining discount rate; see, e.g., Cropper and Laibson (1999).

¹³As a practical matter, I calculate WTP using a finite horizon of 500 years. After some 200 years the world will likely exhaust the economically recoverable stocks of fossil fuels, so that GHG concentrations will fall. In addition, so many other economic and social changes are likely that the relevance of applying CRRA expected utility over more than a few hundred years is questionable.

of τ , the abatement-induced limit on any increase in temperature at the horizon H . Clearly the amount and cost of abatement needed will decrease as τ is made larger, so I consider a stringent abatement policy to be one for which τ is “low,” which I take to be at or below the expected value of ΔT under a business-as-usual (BAU) scenario, i.e., about 3°C.

I examine whether the fitted displaced gamma distributions for ΔT and γ , along with “reasonable” values of the remaining economic parameters can yield values of $w^*(\tau)$ greater than 2 or 3% for $\tau \approx 3^\circ\text{C}$. I also explore “tail effects” by transforming the distributions to increase skewness or variance while keeping the other moments fixed, and calculating the resulting change in $w^*(\tau)$. Finally, to explore the case for a stringent abatement policy, I focus on conservative parameter assumptions, in the sense of leading to a higher WTP.

3 Temperature Change.

The IPCC (2007a) surveyed 22 scientific studies of *climate sensitivity*, the increase in temperature that would result from an anthropomorphic doubling of the atmospheric CO₂e concentration. Given that a doubling (relative to the pre-industrial level) by the end of the century is the IPCC’s consensus prediction, I treat climate sensitivity as a rough proxy for ΔT a century from now. Each of the studies surveyed provided both a point estimate and information about the uncertainty around that estimate, such as confidence intervals and/or probability distributions. The IPCC translated these results into a standardized form so that they could be compared, created graphs with multiple distributions implied by groups of studies, and estimated that the studies implied an expected value of 2.5°C to 3.0°C for climate sensitivity. How one aggregates the results of these studies depends on beliefs about the underlying models and data. Although this likely overestimates the size of the tails, I will assume that the studies used the same data but different models, and average the results. This is more or less what Weitzman (2009) did, and my estimates of the tails from the aggregation of these studies are close to (but slightly lower) than his. To be conservative, I use his estimate of a 17% probability that a doubling of the CO₂e concentration would lead to a mean temperature increase of 4.5°C or more, a 5% probability of a temperature increase

of 7.0°C or more, and a 1% probability of a temperature increase of 10.0°C or more.

I fit a displaced gamma distribution to these summary numbers. Letting θ be the displacement parameter, the distribution is given by:

$$f(x; r, \lambda, \theta) = \frac{\lambda^r}{\Gamma(r)} (x - \theta)^{r-1} e^{-\lambda(x-\theta)} \quad , \quad x \geq \theta \quad , \quad (13)$$

where $\Gamma(r) = \int_0^\infty s^{r-1} e^{-s} ds$ is the Gamma function. The moment generating function is:

$$M_x(t) = \mathcal{E}(e^{tx}) = \left(\frac{\lambda}{\lambda - t} \right)^r e^{t\theta} \quad (14)$$

Thus the mean, variance and skewness (around the mean) are $\mathcal{E}(x) = r/\lambda + \theta$, $\mathcal{V}(x) = r/\lambda^2$, and $\mathcal{S}(x) = 2r/\lambda^3$ respectively.

Fitting $f(x; r, \lambda, \theta)$ to a mean of 3°C, and the 5% and 1% points at 7°C and 10°C respectively yields $r = 3.8$, $\lambda = 0.92$, and $\theta = -1.13$. The distribution is shown in Figure 1. It has a variance and skewness around the mean of 4.49 and 9.76 respectively. Note that this distribution implies that there is a small (2.9 percent) probability that a doubling of the CO₂e concentration will lead to a *reduction* in mean temperature, and indeed this possibility is consistent with several of the scientific studies. The distribution also implies that the probability of a temperature increase of 4.5°C or greater is 21%.

Later I will change the mean, variance and skewness, while keeping the other two moments fixed. Denote the scaling factors by α_M , α_V , and α_S (so setting $\alpha_S = 1.5$ increases the skewness by 50%). Using the equations for the moments, to change the skewness by a factor of α_S while keeping the mean and variance fixed, replace r , λ , and θ with $r_1 = r/\alpha_S^2$, $\lambda_1 = \lambda/\alpha_S$, and $\theta_1 = \theta + (1 - 1/\alpha_S)r/\lambda$. Likewise, to change the variance by a factor α_V , set $r_1 = \alpha_V^3 r$, $\lambda_1 = \alpha_V \lambda$, and $\theta_1 = \theta + (1 - \alpha_V^2)r/\lambda$. To change the mean by α_M , keep r and λ fixed but set $\theta_1 = \alpha_M \theta + (\alpha_M - 1)r/\lambda$, which shifts the distribution to the right or left.

Recall that the distribution for ΔT pertains to a point in time, H , and I assume that temperature follows the trajectory of eqn. (1), so that $\Delta T_t \rightarrow 2\Delta T_H$ as t gets large. This is illustrated in Figure 2, which shows a trajectory for ΔT when it is unconstrained (and ΔT_H happens to equal 5°C), and when it is constrained so that $\Delta T_H \leq \tau = 3^\circ\text{C}$. Note that even when constrained, ΔT_H is a random variable and (unless $\tau = 0$) will be less than τ with probability 1; in Figure 2 it happens to be 2.5°C. If $\tau = 0$, then $\Delta T = 0$ for all t .

4 Economic Impact.

What would be the economic impact (broadly construed) of a temperature increase of 7°C or greater? One might answer, as Stern (2007, 2008) does, that we simply do not (and cannot) know, because we have had no experience with this extent of warming, and there are no models that can say much about the impact on production, migration, disease prevalence, and a host of other relevant factors. Of course we could say the same thing about the probabilities of temperature increases of 7°C or more, which are also outside the range of the climate science models behind the studies surveyed by the IPCC. This is essentially the argument made by Weitzman (2009a), but in terms of underlying “structural uncertainty” that can never be resolved even as more data arrive over the coming decades. But if large temperature increases are what really matter, this gives us no handle on policy.

Instead, I take IAMs and related models of economic impact at face value and treat them analogously to the climate science models. These models yield a rough consensus regarding possible economic impacts: for temperature increases up to 4°C , the “most likely” impact is from 1% to at most 5% of GDP.¹⁴ Of interest is the outer tail of the distribution for this impact. There is some chance that a temperature increase of 3°C or 4°C would have a much larger impact, and we want to know how that affects WTP.

At issue is the value of γ in eqn. (2). Different IAMs and other economic studies suggest different values for this parameter, and although there are no estimates of confidence intervals (that I am aware of), intervals can be inferred from some of the variation in the suggested values. I therefore treat this parameter as stochastic and distributed as gamma, as in eqn. (13). I further assume that γ and ΔT are independently distributed, which is realistic given that they are governed by completely different physical/economic processes.

Based on its own survey of impact estimates from four IAMs, the IPCC (2007b) concludes that “global mean losses could be 1–5% of GDP for 4°C of warming.”¹⁵ In addition, Dietz

¹⁴This consensus might arise from the use of similar ad hoc damage functions in various IAMs.

¹⁵The IAMs surveyed by the IPCC include Hope (2006), Mendelsohn et al (1998), Nordhaus and Boyer (2000), and Tol (2002). For a recent overview of economic impact studies, see Tol (2009).

and Stern (2008) provide a graphical summary of damage estimates from several IAMs, which yield a range of 0.5% to 2% of lost GDP for $\Delta T = 3^\circ\text{C}$, and 1% to 8% of lost GDP for $\Delta T = 5^\circ\text{C}$. I treat these ranges as “most likely” outcomes, and use the IPCC’s definition of “most likely” to mean a 66 to 90-percent confidence interval. Using the IPCC range and, to be conservative, assuming it applies to a 66-percent confidence interval, I take the mean loss for $\Delta T = 4^\circ\text{C}$ to be 3% of GDP, and the 17-percent and 83-percent confidence points to be 1% of GDP and 5% of GDP respectively. These three numbers apply to the value of β in eqn. (3), but they are easily translated into corresponding numbers for γ in eqn. (2). From eqn. (5), $\gamma = 1.79\beta\Delta T/H$. Thus the mean, 17-percent, and 83-percent values for γ are, respectively, $\bar{\gamma} = .0001363$, $\gamma_1 = .0000450$, and $\gamma_2 = .0002295$.¹⁶

Using these three numbers to fit a 3-parameter displaced gamma distribution for γ yields $r_g = 4.5$, $\lambda_g = 21,341$, and $\theta_g = \bar{\gamma} - r_g/\lambda_g = -.0000746$. This distribution is shown in Figure 3. For comparison, I also fit the distribution assuming the 1% to 5% loss of GDP for $\Delta T = 4^\circ\text{C}$ represents a 90-percent confidence interval; it is also shown in Figure 3.

5 Willingness to Pay.

I assume that by giving up a fraction $w(\tau)$ of consumption now and throughout the future, society can ensure that at time H , ΔT_H will not exceed τ . Specifically, the distribution for ΔT is cut off at τ and rescaled to integrate to 1.¹⁷ Using the CRRA utility function of eqn. (6) and the growth rate of consumption given by eqn. (2), $w^*(\tau)$ is the *maximum* fraction of consumption society would sacrifice to keep $\Delta T_H \leq \tau$. As explained in Section 2, $w^*(\tau)$ is found by equating the social welfare functions $W_1(\tau)$ and W_2 of eqns. (11) and (12).

Given the distributions $f(\Delta T)$ and $g(\gamma)$ for ΔT and γ respectively, denote by $M_\tau(t)$ and $M_\infty(t)$ the time- t expectations

$$M_\tau(t) = \frac{1}{F(\tau)} \int_{\theta_T}^{\tau} \int_{\theta_\gamma}^{\infty} e^{\tilde{\omega} - \tilde{\rho}t - \tilde{\omega}(1/2)^{t/H}} f(\Delta T)g(\gamma)d\Delta Td\gamma \quad (15)$$

¹⁶If L is the loss of GDP corresponding to ΔT , $1 - L = \exp[-\beta(\Delta T)^2] = \exp[-.557\gamma H\Delta T]$. $H = 100$ and $\Delta T = 4^\circ\text{C}$, so $.97 = e^{-223.5\bar{\gamma}}$, $.99 = e^{-223.5\gamma_1}$, and $.95 = e^{-223.5\gamma_2}$. Using instead the 4.5% midpoint of the 1% to 8% range of lost GDP for $\Delta T = 5^\circ\text{C}$ from Dietz and Stern (2008), we would have $\bar{\gamma} = .000165$.

¹⁷The expected value of ΔT_H is $\mathcal{E}(\Delta T | \tau) = (\int_0^\tau xf(x)dx) / (\int_0^\tau f(x)dx) < \tau$.

and

$$M_\infty(t) = \int_{\theta_T}^{\infty} \int_{\theta_\gamma}^{\infty} e^{\tilde{\omega} - \tilde{\rho}t - \tilde{\omega}(1/2)^{t/H}} f(\Delta T) g(\gamma) d\Delta T d\gamma, \quad (16)$$

where $\tilde{\rho}$ and $\tilde{\omega}$ are given by eqns. (9) and (10), θ_T and θ_γ are the lower limits on the distributions for ΔT and γ , and $F(\tau) = \int_{\theta_T}^{\tau} f(\Delta T) d\Delta T$. Thus $W_1(\tau)$ and W_2 are:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} \int_0^{\infty} M_\tau(t) dt \equiv \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} G_\tau \quad (17)$$

and

$$W_2 = \frac{1}{1 - \eta} \int_0^{\infty} M_\infty(t) dt \equiv \frac{1}{1 - \eta} G_\infty. \quad (18)$$

Setting $W_1(\tau)$ equal to W_2 , WTP is given by:

$$w^*(\tau) = 1 - [G_\infty / G_\tau]^{\frac{1}{1-\eta}}. \quad (19)$$

The solution for $w^*(\tau)$ depends on the distributions for ΔT and γ , the horizon H , and the parameters η , g_0 , and δ . It is useful to determine how w^* varies with τ ; the cost of abatement is a decreasing function of τ , so given estimates of that cost, one could use these results to determine abatement targets.

5.1 Parameter Values.

As we will see, WTP depends strongly on the values for the index of relative risk aversion η , the rate of time discount δ , and the base level real growth rate g_0 . The finance and macroeconomics literature has estimates of η ranging from 1.5 to 6, and estimates of δ ranging from .01 to .04. The growth rate g , measured directly from historical data, is in the range of .02 to .025. Thus the Ramsey rule puts the consumption discount rate in the range of 3% to over 10%, which can be viewed as close to a private return on investment (PRI). Indeed, most estimates of η and δ are based on investment and/or short-run consumption and savings behavior. The social discount rate (SDR) can differ considerably from the PRI, especially for public investments involving long time horizons and strong externalities. It has been argued, for example, that for intergenerational comparisons δ should be close to zero, on the grounds that there is no reason why society should value the well-being of our

great-grandchildren less than our own. Likewise, while values of η well above 2 may be consistent with the (relatively short-horizon) behavior of investors, we might apply lower values to welfare comparisons involving future generations.¹⁸

Putting aside debates over the “correct” values of η and δ , I want to determine whether current assessments of uncertainty over temperature change and economic impact generate a high WTP and thus justify the immediate adoption of a stringent abatement policy. I will therefore stack the deck, so to speak, in favor of our great-grandchildren and use relatively low values of η and δ : around 2 for η and 0 for δ . Also, WTP is a decreasing function of the base growth rate g_0 in eqn. (7), so I will use a range of .015 to .025 for that parameter.

5.2 No Uncertainty.

Removing uncertainty provides some intuition for the determinants of WTP and its dependence on some of the parameters. If the trajectory for ΔT and the impact of that trajectory on economic growth were both known with certainty, eqns. (17) and (18) would simplify to:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} \int_0^\infty e^{\omega - \rho t - \omega(1/2)^{t/H}} dt , \quad (20)$$

and

$$W_2 = \frac{1}{1 - \eta} \int_0^\infty e^{\omega_\tau - \rho t - \omega_\tau(1/2)^{t/H}} dt , \quad (21)$$

where $\omega = 2(\eta - 1)\bar{\gamma}H\Delta T_H/\ln(1/2)$ and $\omega_\tau = 2(\eta - 1)\bar{\gamma}\tau/\ln(1/2)$. (I am using the mean, $\bar{\gamma}$, as the certainty-equivalent value of γ .)

I calculate the WTP to keep ΔT zero for all time, i.e., $w^*(0)$, over a range of values for ΔT at the horizon $H = 100$. For this exercise, I set $\eta = 2$, $\delta = 0$, and $g = .015$, $.020$, and $.025$. The results are shown in Figure 4. The graph says that if, for example, $\Delta T_H = 6^\circ\text{C}$ and $g_0 = .02$, $w^*(0)$ is about .022, i.e., society should be willing to give about 2.2% of current and future consumption to keep ΔT at zero instead of 6°C .¹⁹ Although $w^*(0)$ is considerably larger if ΔT_H is *known* to be 8°C or more, such temperature outcomes have low probability.

¹⁸For arguments in favor of low values for η and δ and low SDRs, see Heal (2009), Stern (2008) and Summers and Zeckhauser (2008). For opposing views, see Nordhaus (2007) and Weitzman (2007).

¹⁹Remember that the “known ΔT ” is not constant, and applies only to time $t = H$. ΔT follows the trajectory given by eqn. (1).

Note that for any known ΔT_H , a lower initial growth rate g_0 implies a higher WTP. The reason is that lowering g_0 lowers the entire trajectory for the consumption discount rate R_t . That rate falls as ΔT increases (and can eventually become negative), but its starting value is $\delta + \eta g_0$. The damages from warming (a falling growth rate as ΔT increases) are initially small, making estimates of WTP highly dependent on the values for δ , η , and g_0 .

5.3 Uncertainty Limited to Temperature Change.

I now turn to the effects of uncertainty over ΔT . I will assume that the loss function is deterministic, with the parameter γ fixed at its mean value $\bar{\gamma} = .0001363$. Eqns. (15) and (16) then simplify as follows:

$$M_\tau(t) = \frac{1}{F(\tau)} \int_{\theta_T}^{\tau} e^{\tilde{\omega} - \tilde{\rho}t - \tilde{\omega}(1/2)^{t/H}} f(\Delta T) d\Delta T \quad (22)$$

and

$$M_\infty(t) = \int_{\theta_T}^{\infty} e^{\tilde{\omega} - \tilde{\rho}t - \tilde{\omega}(1/2)^{t/H}} f(\Delta T) d\Delta T \quad , \quad (23)$$

where now $\bar{\gamma}$ replaces γ in $\tilde{\rho}$ and $\tilde{\omega}$.

Figure 5 shows $w^*(\tau)$ for $\delta = 0$, $\eta = 2$, and $g_0 = .015$, $.020$, and $.025$. Observe that WTP is below 2%, even for $\tau = 0$, and is closer to 1% if $g_0 = .020$ or $.025$. A feasible (i.e., attainable using a realistic abatement policy) value for τ is probably around 2°C, which makes WTP considerably smaller.

Table 1 shows $w^*(\tau)$ for several values of τ , using the base distribution for ΔT shown in Figure 1, with $\delta = 0$, $\eta = 2$, and $g_0 = .02$. (The loss function parameter γ is again fixed at $\bar{\gamma} = .0001363$.) The first column duplicates the low WTP numbers shown in Figure 5. In the next two columns the skewness and variance of the distribution for ΔT is increased by 50%, in each case holding the other two moments fixed. Increasing the skewness *reduces* $w^*(\tau)$ for $\tau < 5^\circ\text{C}$, because it pushes some of the probability mass from the right to the left tail. For $\tau = 7^\circ\text{C}$ or more, $w^*(\tau)$ is increased, but only modestly, because the probability of a ΔT of 7°C or more is still very low. A 50% increase in the variance of the distribution increases $w^*(\tau)$ for all values of τ , but only modestly.

Table 1: WTP, only ΔT Stochastic

τ	Base Case	$S = 1.5S_0$	$V = 1.5V_0$	$\gamma = .0002726$
0	.0113	.0106	.0126	.0232
1	.0092	.0084	.0107	.0190
3	.0053	.0049	.0069	.0112
5	.0026	.0026	.0038	.0056
7	.0011	.0013	.0017	.0024
10	.0002	.0004	.0003	.0005

Note: Each entry is $w^*(\tau)$, fraction of consumption society would sacrifice to ensure that $\Delta T_H \leq \tau$. $H = 100$ years, $\delta = 0$, $\eta = 2$, $g_0 = .02$.

The last column of Table 1 shows $w^*(\tau)$ for the original distribution of ΔT , but a doubling of the loss function parameter γ . This has a substantial effect on the WTP, roughly doubling all of the base case numbers. But $w^*(0)$ is still only about 2.3%.

5.4 Uncertainty Over Temperature and Economic Impact.

I now allow for uncertainty over both ΔT and the impact parameter γ , using the calibrated distributions for each. WTP is now given by eqns. (15) to (19). The calculated values of WTP are shown in Figure 6 for $\delta = 0$, $\eta = 2$, and $g_0 = .015$, $.020$, and $.025$. Note that if g_0 is $.02$ or greater, WTP is always less than 1.2%, even for $\tau = 0$. To obtain a WTP at or above 2% requires an initial growth rate of only $.015$ or a lower value of η . The figure also shows the WTP for $\eta = 1.5$ and $g_0 = .02$; now $w^*(0)$ reaches 3.5%.

Figure 7 shows the dependence of WTP on the index of risk aversion, η . It plots $w^*(3)$, i.e., the WTP to ensure $\Delta T_H \leq 3^\circ\text{C}$ at $H = 100$ years, for $g_0 = .02$. Although $w^*(3)$ is below 2% for moderate values of η , it comes close to 6% if η is reduced to 1 (the value used in Stern (2007)). The reason is that while future utility is not discounted (because $\delta = 0$), future consumption is implicitly discounted at the initial rate ηg_0 . If η (or g_0) is made smaller, potential losses of future consumption have a larger impact on WTP. Finally, Figure 7 also shows that discounting future utility, even at a very low rate, will considerably reduce WTP. If δ is increased to $.01$, $w^*(3)$ is again below 2% for all values of η .

I obtained large values of WTP only for fairly extreme combinations of parameter values. However, these results are based on distributions for ΔT and γ that were calibrated to studies in the IPCC's 2007 report and concurrent economic studies, and those studies were done several years prior to 2007. Some more recent studies indicate that “most likely” values for ΔT in 2100 might be higher than the 1.0°C to 4.5°C range given by the IPCC. For example, a recent report by Sokolov et al (2009) suggests an expected value for ΔT in 2100 of around 4 to 5°C, as opposed to the 3.0°C expected value that I used.

Suppose the distribution for ΔT_H is correct, but warming occurs faster so it now applies to a shortened horizon of $H = 75$ years. Figure 8 duplicates Figure 7 except that $H = 75$. Observe that if $\delta = 0$ and η is close to 1, $w^*(3)$ reaches 8%. (However, $w^*(3)$ is much lower if $\delta = .01$.) Alternatively, suppose the distribution for ΔT_H has a mean of 5°C, corresponding to the upper end of the 4 to 5°C range in Sokolov et al (2009). Figure 9 duplicates Figure 7 except the mean of ΔT_H is increased to 5°C, with the other moments of the distribution unchanged (and $H = 100$ years). Now if $\delta = 0$ and η is below 1.5, $w^*(3)$ is above 3%, and reaches 10% if $\eta = 1$. Thus there are parameter values and plausible distributions for ΔT that yield a large WTP, but that are outside of the current consensus range.

5.5 Policy Implications.

The policy implications of these results are stark. For temperature and impact distributions based on the IPCC and “conservative” parameter values (e.g., $\delta = 0$, $\eta = 2$, and $g_0 = .02$), WTP to prevent *any* increase in temperature is around 2% or less. And if the policy objective is to ensure that ΔT in 100 years does not exceed its expected value of 3°C (a much more feasible objective), WTP is lower still.

There are two reasons for these results. First, there is limited weight in the tails of the distributions for ΔT and γ . The distribution calibrated for ΔT implies a 21% probability of $\Delta T \geq 4.5^\circ\text{C}$ in 100 years, and a 5% probability of $\Delta T \geq 7.0^\circ\text{C}$, numbers consistent with the climate sensitivity studies surveyed by the IPCC. Likewise, the calibrated distribution for γ implies a 17% probability of $\gamma \geq .00023$, also consistent with the IPCC and other surveys. A realization in which, say, $\Delta T = 4.5^\circ\text{C}$ and $\gamma = .00023$ would imply that GDP and

consumption in 100 years would be 5.7 percent lower than with no increase in temperature.²⁰ However, the probability of $\Delta T \geq 4.5^\circ\text{C}$ and $\gamma \geq .00023$ is only about 3.6%. An even more extreme outcome in which $\Delta T = 7^\circ\text{C}$ (and $\gamma = .00023$) would imply about a 9 percent loss of GDP in 100 years, but the probability of an outcome this bad or worse is only 0.9%.

Second, even if $\delta = 0$, the implicit discounting of consumption is significant. The initial consumption discount rate is $\rho_0 = \eta g_0$, which is at least .03 if $\eta = 2$. And a (low-probability) 5.7 or 9 percent loss of GDP in 100 years would involve much smaller losses in earlier years.

Although these estimates of WTP do not support the immediate adoption of a stringent GHG abatement policy, they do not imply that no abatement is optimal. For example, 2% of GDP is in the range of cost estimates for compliance with the Kyoto Protocol.²¹ Taking the U.S. in isolation, a WTP of 2% amounts to about \$300 billion per year, a rather substantial amount for GHG abatement. And if, e.g., $w^*(3) = .01$, a \$150 billion per year expenditure on abatement would be justified if it would indeed limit warming to 3°C .

6 Conclusions.

I have approached climate policy analysis from the point of view of a simple measure of “willingness to pay”: the fraction of consumption $w^*(\tau)$ that society would sacrifice to ensure that any increase in temperature at a future point is limited to τ . This avoids having to make projections of GHG emissions and atmospheric concentrations, or estimate abatement costs. Instead I could focus directly on uncertainties over temperature change and over the economic impact of higher temperatures. Also, I modeled economic impact as a relationship between temperature change and the growth rate of GDP as opposed to its level. Using information on the distributions for temperature change and economic impact from studies assembled by the IPCC and from recent IAMs (the current “state of knowledge” regarding warming and its impact), I fit displaced gamma distributions for ΔT and an impact parameter γ . I then examined whether “reasonable” values for the remaining parameters could yield values

²⁰If $\gamma = .00023$ and $\Delta T = 4.5^\circ\text{C}$, $\beta = \gamma H / 1.89 \Delta T = .00270$, and from eqn. (3), $L = e^{-\beta(\Delta T)^2} = .947$.

²¹See the survey of cost studies by the Energy Information Administration (1998), and the more recent country cost studies surveyed in IPCC (2007c).

of $w^*(\tau)$ above 2% or 3% for small values of τ , which might support stringent abatement, and found that for the most part they could not.

For “conservative” parameter values, e.g., $\delta = 0$, $\eta = 2$, and $g_0 = .015$ or $.02$, WTP to prevent *any* increase in temperature is only around 2%, and is well below 2% if the objective is to keep ΔT in 100 years below its expected value of 3°C. Given what we know about the distributions for temperature change and its impact, it is difficult to obtain a large WTP unless η is reduced to 1.5 or less, or we assume warming will occur at a more accelerated rate than the IPCC projects. There are two reasons for these results: limited weight in the tails of the distributions for ΔT and γ , and the effect of consumption discounting.

It is an understatement to say that caveats are in order. First, although I have incorporated what I believe to be the current consensus on the distributions for temperature change and its impact, this consensus may be wrong, especially with respect to the tails of the distributions. Indeed, some recent studies suggest that warming could be greater and/or more rapid than the IPCC suggests. We have no historical or experimental data from which to assess the likelihood of a ΔT above 5°C, never mind its economic impact, and one could argue á la Weitzman (2009) that we will never have sufficient data because the distributions are fat-tailed, implying a WTP of 100% (or at least something much larger than 2%). In addition, the loss function of eqn. (2) is linear, and a convex relationship between ΔT and the growth rate g_t may be more realistic.

The real debate among economists is not so much over the need for some kind of GHG abatement policy, but rather whether a stringent policy is needed now, or instead abatement should begin slowly. My results are consistent with beginning slowly. In addition, beginning slowly has other virtues. It is likely to be dynamically efficient because of discounting (most damages will occur in the distant future) and because of the likelihood that technological change will reduce the cost of abatement over time. Also, there is an “option value” to waiting for more information. Over the next decade or two we may learn much more about climate sensitivity, the economic impact of warming, and the cost of abatement, in part from ongoing research, and in part from the accumulation of additional data.

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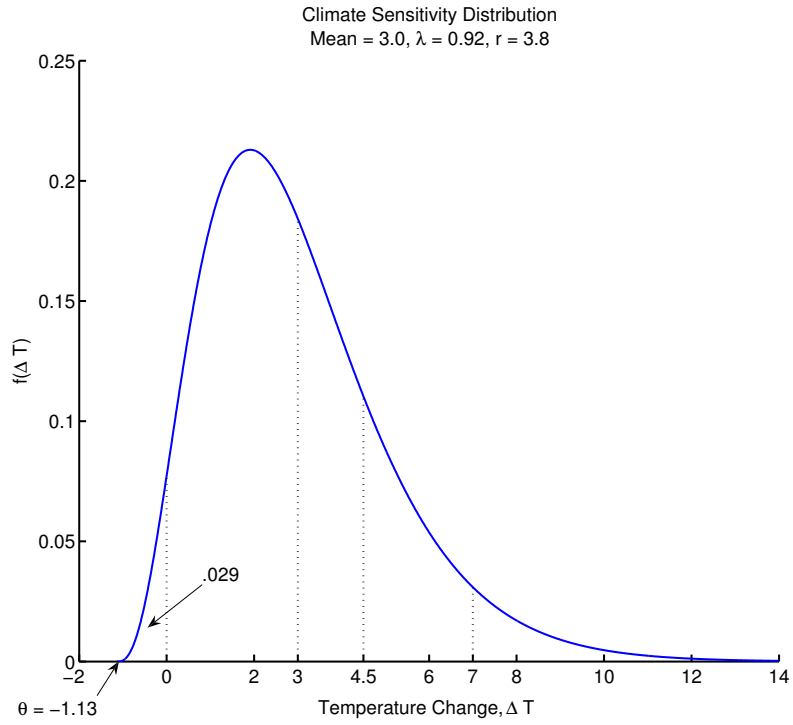


Figure 1: Base Distribution for Temperature Change.

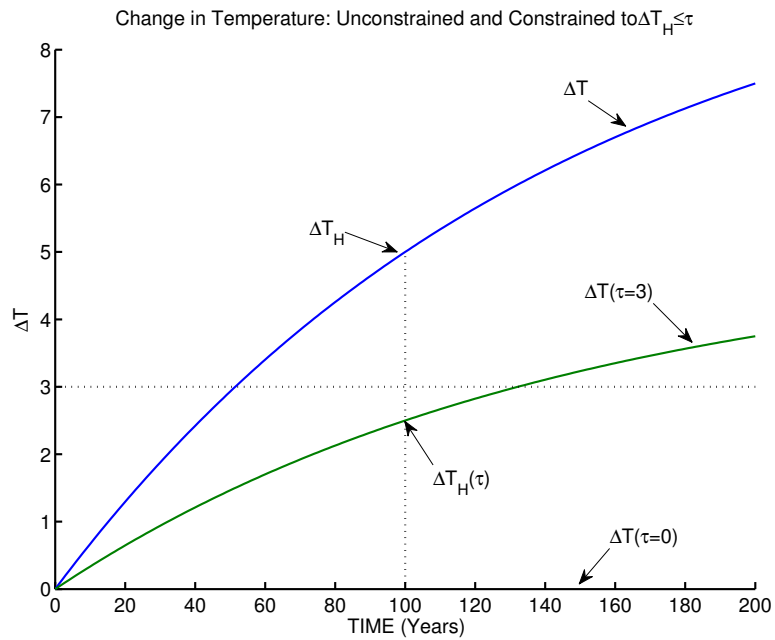


Figure 2: Temperature Change: Unconstrained and Constrained So $\Delta T_H \leq \tau$

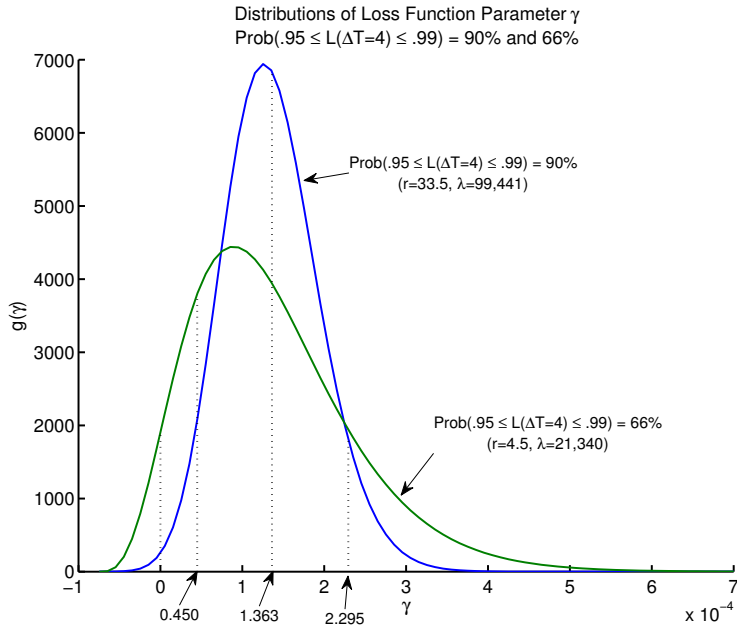


Figure 3: Distributions for Loss Function Parameter γ

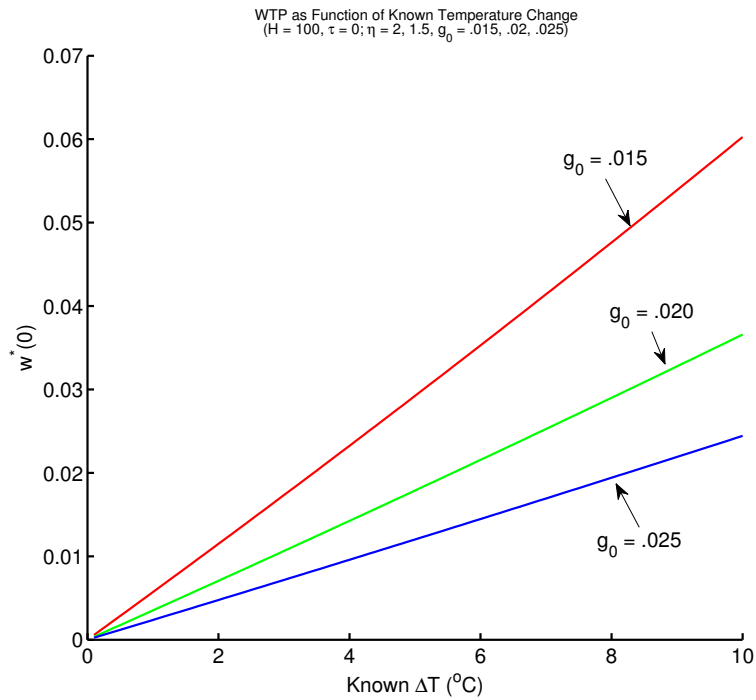


Figure 4: WTP, Known Temperature Change, $\eta = 2$, $g_0 = .015, .020, .025$, and $\delta = 0$

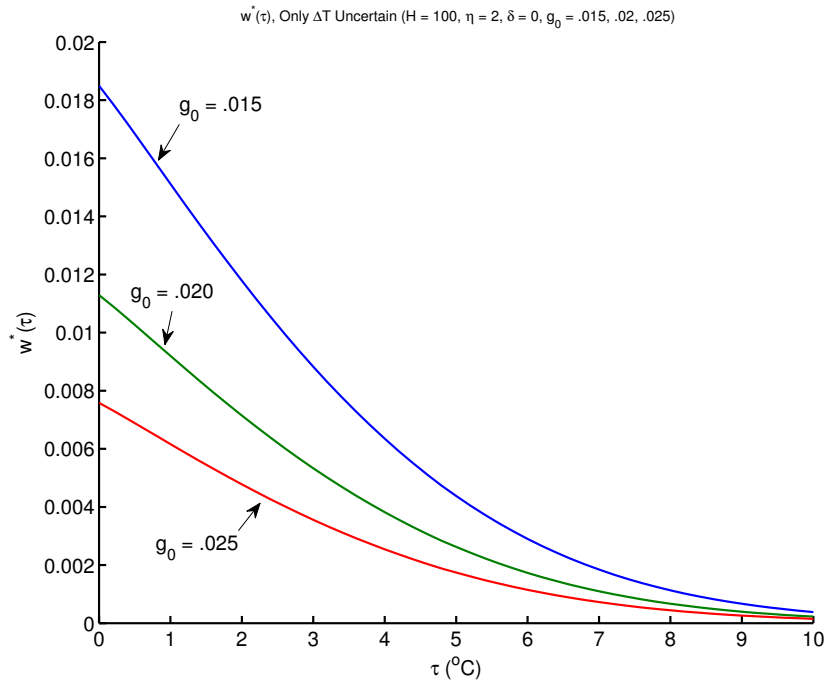


Figure 5: WTP, Only ΔT Uncertain, $\eta = 2, \delta = 0$

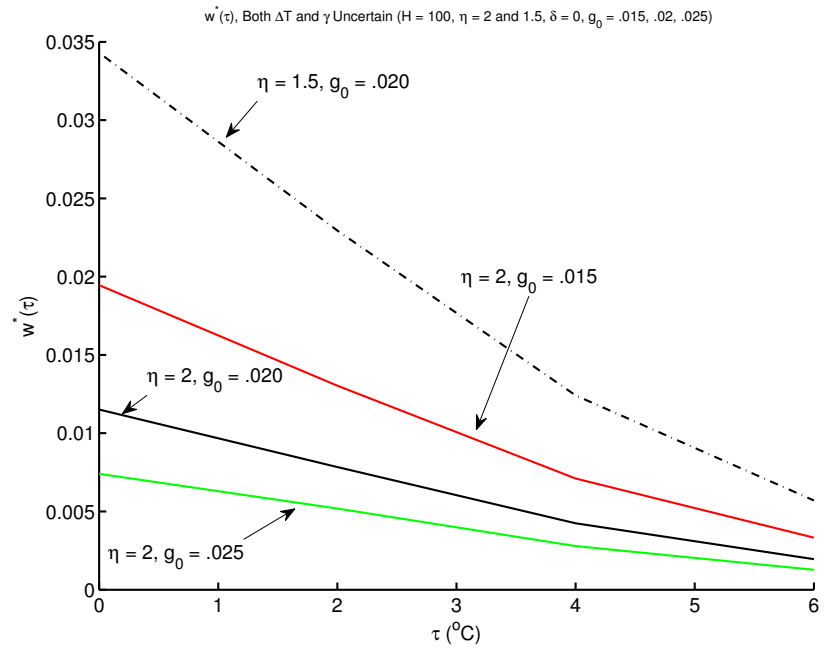


Figure 6: WTP, Both ΔT and γ Uncertain. $\eta = 2$ and $1.5, g_0 = .015, .020, .025$, and $\delta = 0$

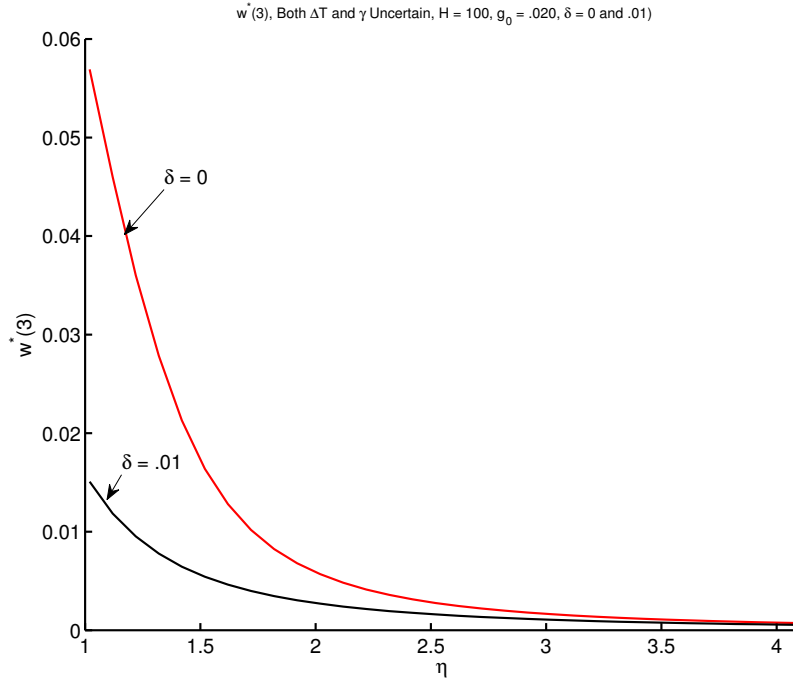


Figure 7: WTP Versus η for $\tau = 3$. $g_0 = .020$ and $\delta = 0$ and $.01$

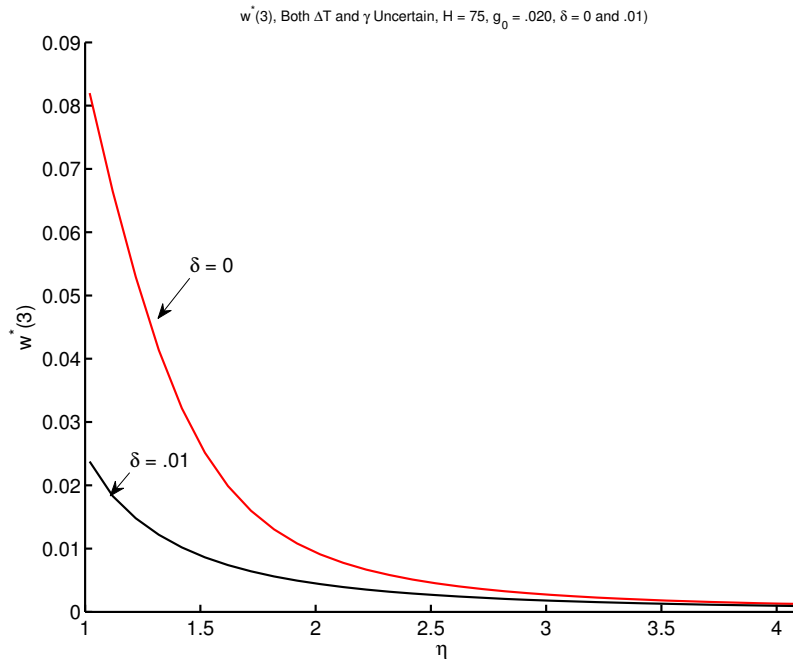


Figure 8: WTP Versus η for $\tau = 3$. $H = 75$, $g_0 = .020$, $\delta = 0$ and $.01$

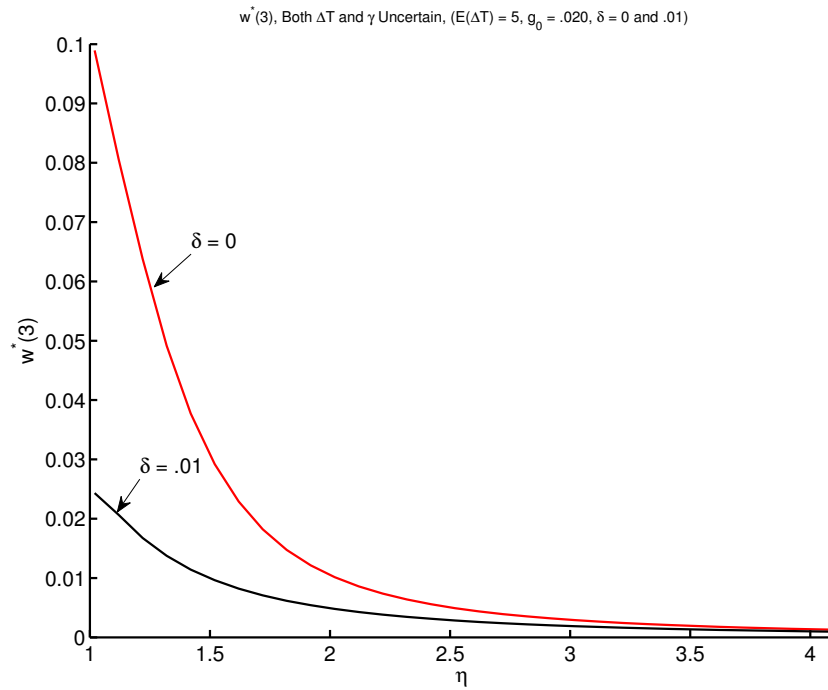


Figure 9: WTP Versus η for $\tau = 3$. $\mathcal{E}(\Delta T_H) = 5^\circ\text{C}$, $H = 100$, $g_0 = .020$, $\delta = 0$ and $.01$