Due Date: Monday, November 8, 2004

12. Now consider another individual with the same preferences as the person analyzed above:

$$V(C_1, C_2) \quad = \quad \log C_1 + [1/(1+\delta)] \log C_2.$$

This individual faces a wage income tax, so that the interest rate at which she can borrow and lend is simply r. Assume that her wage income in periods 1 and 2 is fixed at Y_1 and Y_2, and that her labor income tax rate in period 1 is τ_1 while that in period 2 is τ_2. Assume that $\tau_1 > \tau_2$, and that both taxes are linear. Further assume that the individual has access to a "tax avoidance technology" that permits wage income to be shifted from period 1 to period 2. If the individual chooses to shift A dollars from period 1 to period 2, where A is between 0 and Y_1, her taxable income in period 1 will be $Y_1 - A$ and that in period 2 will be $Y_2 + A$. Using the tax avoidance technology is costly; the cost of shifting A dollars is $\beta(A)$. This cost can be viewed as the legal and administrative fees associated with tax avoidance, and it must be paid in period 1.

(a) Find the lifetime budget constraint for this individual, recognizing both the impact of tax avoidance on income net of taxes, and the cost of tax avoidance.

(b) Now obtain first order conditions for the optimal choice of A. Does the optimal level of A depend on the utility function? Explain why or why not.

(c) Consider the case in which $\beta(A) = \gamma A^2$, and assume that $r = 0$. Obtain a formula for A as a function of the tax rates in the two periods, and compute the elasticity of tax avoidance (A) with respect to $(1-\tau_1)$. Briefly describe the implications for the impact of tax changes on revenue.

2. The design of targeted saving incentive schemes is a subject of constant policy debate. Consider an economy in which consumers live for two periods, and have preferences given by: $U = \log C_1 + \log C_2$. Each consumer receives an endowment of 1 at the beginning of the first period of life. The pretax return available to consumers is 200%, so someone who saves \$1 receives \$3 at the beginning of the next period (the effective "period" is long!). The income tax rate is 50%. The government uses tax revenue to pay tribute to a foreign power.

(a) Find the optimal lifetime consumption profile of this consumer. Contrast this with the optimal choice if there were no income tax.

(b) Now assume that the government introduces a "retirement saving program" that allows each consumer to save up to .20 in the first period of life in a tax-free account. Sketch the consumer's budget constraint when the income tax rate is 50%, and there is no saving program, and again when the program is in place.

(c) Find the consumer's optimal lifetime consumption plan in the case with the retirement saving program. Explain how the introduction of this program affects first-period saving.

(d) When a new government takes power, it changes the structure of the saving incentive scheme to tax investment income on the first .50 of saving, but to exempt any saving in excess of 0.5 from taxation. Sketch the budget set associated with this policy, and find the consumer's optimal consumption profile. Can you explain the difference in the impact of this policy and that in (b)?

(e) In (d), if the government raises the threshold for tax-exempt saving from .50 to .51, how would this policy change affect personal saving? What would it do to national saving?

3. This question, as well as the next two, relate to the neoclassical growth model with taxes that we discussed in class in October 24. In the representative agent economy discussed in class, make whatever additional assumptions you feel are necessary to demonstrate the following two theorems:

(a) Any competitive equilibrium with taxes $\{\tau^l_t, \tau^k_t\}$ with $\tau^k_0 = 0$ is equivalent to a competitive equilibrium with taxes on consumption and labor income, in the sense that all allocations are the same in the two equilibria.

(b) Any competitive equilibrium with taxes $\{\tau^l_t, \tau^k_t\}$ with $\tau^k_0 = 0$ is equivalent to a competitive equilibrium where all taxes are imposed on the production sector.

4. Consider, again, the representative agent model discussed in class. Show that any allocations that satisfy the implementability (ImC) and feasibility (F) constraints are equivalent to equilibrium allocations. To demonstrate this, you need to prove two things:

(a) Any $\{c_t, k_t, l_t\}$ that constitute a competitive equilibrium satisfy (ImC) and (F); and

(b) For any $\{c_t, k_t, l_t\}$ that satisfy (ImC) and (F), there exist prices $\{r_t, w_t\}$ and a time series of government debt $\{b_t\}$ such that $\{c_t, k_t, l_t, r_t, w_t, b_t\}$ is a competitive equilibrium.

W proved part of this theorem in class. Be sure that you explain the logic behind this part; you do not need to recopy it. You need to develop and write out the proof for the other part.

5. Suppose that the neoclassical growth model discussed in class is modified to consist of two types of infinitely lived agents, workers and capitalists. Capitalists have no labor income, but have an initial endowment of capital k_0. They invest the capital, make decisions about how much to save and to borrow, and they live off the interest income. Workers have no endowment of capital and have 1 unit of labor endowment in each period. Lifetime utility for capitalists is given by $U^k = \Sigma\beta^t u(c^k_t)$ and for workers by $U^w = \Sigma\beta^t u(c^w_t, l^w_t)$. Let k denote capitalists and w denote workers. The social welfare function is $W = \varphi U^k + (1 - \varphi) U^w$ for some $\varphi \geq 0$. Prove that for any φ taxes on capital should be zero in the long run. Note that this result holds even when the utility of the capitalist does not enter the social welfare function ($\varphi = 0$).