Due Date: Friday, September 19, 2003

1. Consider an economy that is populated by two types of consumers. The first has preferences over a private good, x, and a non-excludable public good, G, that are given by:

$$u = \log x + \log G.$$

The second group of consumers has similar preferences given by

$$u = \log x + \beta \log G$$

for $0 =< \beta < 1$. The argument G in both types' utility function is $G = \Sigma_i g_i$ where the sum is over all consumers of both types. Assume that there are N consumers of each type in the economy. Type 1 consumers have an initial endowment of one unit of the private good, while type 2 consumers have an initial endowment of 0.5 units of the private good. One unit of the private good can be costlessly transformed into one unit of the public good (and vice versa),

a) Find the decision rule that each type 1 and each type 2 consumer will use in trying to decide how much to spend on the public good. In a symmetric Nash equilibrium in which all type 1 consumers contribute the same amount to the public good, and all type 2 consumers contribute the same amount to the public good, how much will the economy spend on public goods?

b) Now assume that there is a social planner who has access to a poll tax (a lump sum tax that is levied at the same rate on everyone in the economy). The revenue from the tax can be used to fund spending on the public good. If the social planner seeks to maximize the sum of utilities in the economy, find the optimal level of the poll tax. How does the level of the public good compare with the level in the private provision equilibrium of (a)?

2. Consider a small open economy similar to the economy in the last problem, but now with three types of consumers. All have preferences given by

$$u = \log x + \log G$$

but the three groups have different endowment levels of the private good. The first group has an endowment of x=1, the second has x=2, and the third has x=3. There are equal numbers of all three groups in the economy. Assume that the world price of G is twice the price of x. If the cost of providing the public good is divided equally across everyone in the economy, and everyone votes to determine the level of G, what level of public good will the political process choose? If your answer depends on N, provide an economic explanation for the source of this dependence.

3. Assume that a consumer's utility function is given by:

$$u(x_1, x_2, x_3) = \beta_1 \log (x_1 - \alpha_1) + \beta_2 \log (x_2 - \alpha_2) + \beta_3 \log x_3$$

and that the consumer faces consumer prices q_1 and q_2, with the price of good three normalized to unity. The consumer's endowment (y) is measured in units of good three.

(a) Find the indirect utility function and expenditure function corresponding to this set of preferences. (This is the famous Stone-Geary utility function, the basis for the linear expenditure system in demand analysis.)

(b) Use your results from (a) to find an analytic expression for the compensating variation. In the special case of $\alpha_1 = \alpha_2 = .50$ and $\beta_1 = \beta_2 = .40$, find the CV associated with a tax reform that changes (q_1, q_2, y) from (1, 1, 5) to (2.0, 1.5, 5). How would you expect the EV to compare with the CV in this case (you do not need to compute the EV, but simply to note whether you believe it would be larger or smaller, and why).

(c) Assuming that the differences between the pre- and post-reform consumer prices in (b) are due only to tax changes, i.e. that producer prices are fixed, compute the government's revenue from the new tax policy.