Designing Optimal Disability Insurance: A Case for Asset Testing

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Abstract

This paper analyzes an implementation of an optimal disability insurance system as a competitive equilibrium with taxes. The problem is modeled as a dynamic mechanism design problem in which disability is unobservable. We show that an asset-tested disability system in which a disability transfer is paid only if an agent has assets below a specified maximum implements the optimum. The logic behind the result is that an agent who plans to falsely claim disability: (a) finds doing so unattractive if he does not adjust his savings, and (b) cannot collect disability insurance if he does adjust his savings in the desired direction (upward). We also evaluate the welfare benefits of asset testing. For a calibrated economy, we numerically compare the optimal system to the best system without asset testing. We find that gains from asset testing are significant and equal to about 0.5 percent of consumption.

1 Introduction

The Social Security Disability Insurance (SSDI) program is one of the largest social insurance programs in the United States. In 2001, the program provided income to more than 6 million individuals, who accounted for 14 percent of Social Security beneficiaries. The program cost $61 billion and constituted 15 percent of Social Security benefits. The size of the program far surpasses spending on unemployment insurance, food stamps, or any other similar program (SSA 2000).

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As in the classic work of Diamond and Mirrlees (1978, 1986), we assume that it is impossible to know whether an individual is truly disabled, and that disability is a permanent state. We then solve a dynamic mechanism design problem and provide theoretical and numerical characterizations of the social optimum.

The first goal of the paper is to find a tax system that implements the optimal allocation. By *implementation* we mean finding a tax system such that a solution to a competitive equilibrium problem with taxes coincides with the optimal solution. We first show that a system conjectured by Diamond and Mirrlees (1978), consisting of a linear tax equal to the intertemporal wedge in the optimal allocation, does not implement the optimum. Then we propose a tax system implementing the optimum: an asset-tested disability program. An asset test is a form of a means test in which a person receives a disability transfer only if his assets are below a specified threshold. The logic behind the result is that an agent who plans to falsely claim disability: (a) finds doing so unattractive if he does not adjust his savings, and (b) cannot collect disability insurance if he does adjust his savings in the desired direction (upward).

We then numerically characterize features of the optimal allocations and welfare gains of asset testing. To evaluate advantages of asset testing, we provide estimates of welfare gain obtained by shifting from the best program without asset testing to the optimal program. The best program without asset testing is equivalent to the solution of the optimal program with hidden savings. The welfare gain of asset testing is thus the difference in welfare between the optimal program with and without hidden savings. In a calibrated model economy, we find a significant welfare gain of using asset testing equal to 0.5 percent of consumption.

Golosov, Kocherlakota, and Tsyvinski (2004) provide a characterization of the optimal allocation in an economy with dynamic, stochastic, private skills. Unlike this paper, their work characterizes an intertemporal wedge but does not derive implementation with a tax system.

The papers most closely related to our work are Albanesi and Sleet (2004) and Kocherlakota (2004a), who also consider a tax-based implementation of a dynamic Mirrlees problem. Albanesi and Sleet (2004) derive an implementation with labor and wealth taxes in an environment with i.i.d. skill shocks. In their environment, wealth summarizes agents' past histories of shocks and allows to define a recursive tax system that depends only on current wealth and effective labor. Their implementation does not work in our setup, as disability is a persistent, in fact, permanent skill shock. Kocherlakota (2004a) allows for a general process for skill shocks and derives an implementation with linear taxes on wealth and arbitrarily nonlinear taxes on the history of effective labor. The optimum in our model can be implemented using taxes similar to the taxes in that paper. That implementation would entail a regressive wealth tax schedule in which an agent who becomes disabled has to pay a high tax while an able agent receives a subsidy for his savings.[1]. Another difference from Albanesi and Sleet (2004) and Kocherlakota (2004a) is that we also focus on the quantitative evaluation of welfare gains of asset testing in a calibrated multi-period model by comparing an optimal system to the best system without asset testing.

This paper also contributes to the study of optimal dynamic social insurance programs (Wang and Williamson (1996) and Hopenhayn and Nicolini (1997)). The focus in these papers was on finding optimal allocations rather than on tax systems implementing them. The difficulty in constructing transfer systems that we highlight in this paper is present in other dynamic mechanism design models, such as models of optimal unemployment insurance. The techniques of implementation in the presence of private savings that we develop in this paper can be used in those settings.

The key to our analysis is that we assume disability is unobservable and permanent. In practice, determining disability status proves to be very difficult. Multiple medical and vocational factors are taken into account when determining whether an individual is eligible for disability benefits. However, even the determination of medical factors is often subjective. In 2001, a share of awards to applicants with difficult-to-verify criteria, such as mental disorders (mainly mental stress and excluding retardation) and diseases of the musculoskeletal system (typically back pain), constituted around 50 percent of total awards. Disability is a fairly permanent state. For example, only less than one percent of those who start receiving disability benefits return to work[2].

The rest of the paper is structured as follows: In the next section, we describe the setup of the model. In Section 3, we provide a theoretical characterization of the optimum. In Section 4, we discuss implementation of the optimum. In Section 5, we provide numerical results. In Section 6, we discuss the robustness of these theoretical results and the role of our assumptions.

[1] If we use this implementation for the economy that we compute in our paper the wealth taxes on disabled range from 55 percent early in life to 10 percent late in life while the subsidy to savings of able ranges from 0.1 percent early in life to 0.5 percent late in life.

[2] A low number of disabled returning to work does not necessarily mean that disability is a permanent state. It could indicate, for example, generosity of benefits. However, a very low number of those returning to work gives us confidence in modelling disability as a permanent state. For a detailed discussion of difficulties in determining disability and the data on the number of people leaving disability, see Bound and Burkhauser (1998).

2 Setup

An agent lives for T periods and has preferences represented by a utility function

$$E\sum_{t=1}^{T}\beta^{t-1}[u(c_t)+v(l_t)],$$

where E denotes expectation operator, $0<\beta<1$, and c_t and l_t denote the period t consumption and labor of an agent. We assume that $u'>0$, $u''<0$, and $v'<0$.

An agent can become disabled in period t, and his skill θ_t is then equal to zero. We assume that disability is an absorbing state and that once disabled, an agent stays disabled for the rest of his life. Skills of able agents evolve deterministically over time.

We use the following notation for probabilities. Let

$$\pi_1 = \Pr\left[\text{able at } t=1\right],$$

$$\pi_t = \Pr\left[\text{able at } t|\text{ able at } t-1\right] \text{ for } t=2,...,T,$$

$$\Pi_{s,t} = \pi_s\cdot ...\cdot\pi_t = \Pr\left[\text{able at } t|\text{ able at } s-1\right],$$

$$\Pi_t = \Pr\left[\text{able at } t\right] \text{ for } t=1,...,T,$$

$$\Pi_0 = 1.$$

Because disability is an absorbing state, we need to keep track only of the agent's age and the age at which he became disabled. We denote consumption of an able agent of age t as c_t, his labor as l_t, and consumption of an agent who became disabled at $s\le t$ by x_t^s.

An agent who was able at $t-1$ learns at the beginning of period t whether or not he has become disabled. The information is private: it is never observed by anybody else. Work effort l_t is also private information. Only effective labor supply $y_t=\theta_t l_t$ is observable to outsiders. If $y_t>0$, an outsider can infer that an agent is able. If $y_t=0$, an outsider cannot tell if the worker is disabled or able but exerting no effort. A disabled worker does not exert effort, as it reduces his utility and $y_t=0$ even if he exerts himself. Let $v(0)=0$ be the utility from exerting no effort.

We consider a setting in which the net interest rate R and the wage w are constant over time and assume that $\beta=\dfrac{1}{1+R}$. An allocation of consumption and labor (c,l,x) is *feasible* if and only if

$$\sum_{t=1}^{t=T} \beta^{t-1} \Pi_t c_t + \sum_{s=1}^{T} \Pi_{s-1} (1-\pi_s) \sum_{t=s}^{T} \beta^{t-1} x_t^s \leq \sum_{t=1}^{t=T} \beta^{t-1} \Pi_t w \theta_t l_t. \tag{1}$$

This condition sates that the expected present value of consumption allocations is smaller than the expected present value of output.

Allocations must respect incentive-compatibility conditions, since disability status is private information. In particular, since disability is an absorbing state and an agent who claims disability would not later claim to be able[3], there are T incentive constraints. These constraints require that in each period the expected utility of working is higher than the utility of claiming disability:

$$\begin{aligned} & [u(c_s) + v(l_s)] + \sum_{t=s+1}^{t=T} \beta^{t-s} \Pi_{s+1,t} [u(c_t) + v(l_t)] \\ & + \sum_{t=s+1}^{T} \Pi_{s+1,t-1} (1-\pi_t) \sum_{i=t}^{T} \beta^{i-s} u\left(x_i^t\right) \\ \geq & \sum_{t=s}^{T} \beta^{t-s} u(x_t^s) \quad \text{for } \forall s, 1 \leq s \leq T, \end{aligned} \tag{2}$$

where $\Pi_{i,k} = 1$ if $i < k$.

A social planner maximizes the expected utility of the representative agent and solves the following programming problem (P):

$$\max_{c,l,x \geq 0} \sum_{t=1}^{t=T} \beta^{t-1} \Pi_t [u(c_t) + v(l_t)] + \sum_{s=1}^{T} \Pi_{s-1} (1-\pi_s) \sum_{t=s}^{T} \beta^{t-1} u(x_t^s)$$

subject to the feasibility (1) and incentive compatibility (2) constraints.

3 Characterizing Pareto Optima

In this section, we provide a theoretical characterization of the optimal allocation.

A useful benchmark is a case in which disability status is perfectly observable. Then a social planner can achieve full insurance. That is, for all t, s $(s \leq t)$, $c_t^* = x_t^{s*} = \bar{c}$, i.e., consumption is constant over time, and consumption of the able and disabled is equalized. The consumption-labor

[3] An agent previously claiming disability and later working reveals that he lied, hence, the planner can prevent such deviation.

margin is also undistorted:

$$-v'\left(l_t^*\right)\frac{1}{\theta_t} = u'(c_t^*)w.$$

We proceed to characterize the optimal solution when disability is unobservable. We call an allocation (c, l, x) *interior* if $l_t > 0$ for all t. This assumption is satisfied if the skill θ_t is sufficiently high. In the rest of the paper, we assume that the optimum is interior. It is easy to show that, in the optimal allocation, incentive constraints in each period and the feasibility constraint holds with equality. Subtracting the first order conditions for x_t^t from those for c_t we can also derive that $c_t > x_t^t$ for all t.

The proposition that follows provides a characterization of the optimal allocation. We show that the consumption-labor margin is undistorted for able agents. This result is reminiscent of a result that in a static environment labor decisions of the highest skilled agent are undistorted (Mirrlees 1976). The intertemporal margin, however, is distorted as in Golosov, Kocherlakota, and Tsyvinski (2003). Any optimal solution has a wedge between a marginal rate of substitution and the interest rate in every period where the agent, if able, has a strictly positive probability of becoming disabled in the following period. After an agent becomes disabled, all uncertainty is resolved, and there is no need to distort his intertemporal decision. Since we assumed that $\beta = \dfrac{1}{1+R}$, the consumption of the disabled is constant. The proof of the proposition summarizing the characterization of the optimum follows from examination of the first order conditions of the planner's problem.

Proposition 1 *Suppose* (c^*, l^*, x^*) *solves* (P)*.*

1. *For each period* t*, the consumption-labor margin of an able agent is not distorted:*

$$-v'\left(l_t^*\right)\frac{1}{\theta_t} = u'(c_t^*)w,$$

2. *For each period* $t < T$*, the inverse Euler equation holds:*

$$\frac{1}{u'(c_t^*)} = \left[\frac{\pi_{t+1}}{u'(c_{t+1}^*)} + \frac{1-\pi_{t+1}}{u'(x_{t+1}^{t+1*})}\right],$$

3. *Consumption of disabled is constant:*

$$x_t^{s*} = x_{t'}^{s*}, \textit{ for } 1 \le s \le t < t' \le T.$$

Suppose the disability status is not perfectly predictable (given that an agent is able). Then we can apply Jensen's Inequality to the inverse Euler equation to prove that intertemporal decisions of an able agent are distorted.

Corollary 1 *Suppose* (c^*, l^*, x^*) *solves* (P). *Then, if the probability of becoming disabled is interior* $(0 < \pi_{t+1} < 1)$,

$$u'(c_t^*) < \pi_{t+1} u'(c_{t+1}^*) + (1 - \pi_{t+1})\, u'(x_{t+1}^{t+1*}). \tag{3}$$

4 Implementation of the Optimum

In this section we propose a tax system that implements the optimal allocation and includes only taxes and transfers similar to those already in the U.S. tax code.

Since the only restrictions on the social planner's problem are incentive compatibility and feasibility, we implicitly allow a very large set of taxes. Because of the generality of taxes, the social planner's allocation can be implemented in multiple ways, the most obvious of which is a direct mechanism. However, the direct mechanism may include taxes that have never been used in practice.

We first illustrate a difficulty in constructing tax systems with an example of a linear savings tax as in Diamond and Mirrlees (1978). This type of implementation is common in the Ramsey literature of optimal taxation (see a review in Chari and Kehoe 1999). We show that such a tax does not implement the optimum, since it cannot prevent agents from overaccumulating assets and falsely claiming disability.

We then propose a tax/transfer system that implements the optimum: an asset-tested disability system. The first feature of this system is that disability transfers depend on the length of pre-disability work history. The second feature is designed to control the negative incentive effects of savings: disability transfers should be asset-tested, that is, paid only to agents who have assets below a pre-specified minimum.[4] Asset-tested programs, such as Medicaid, Temporary Assistance to Needy Families (TANF), and many others, are used widely in the U.S. social insurance system[5].

First, we formally define competitive equilibrium with general taxes.

[4] Recent empirical evidence supports our argument that persons who falsely claim disability have higher savings than disabled persons. A comprehensive study of disability applicants and recipients by Benitez-Silva, Buchinsky, and Rust (2004) finds (see Table 4 in their paper) that non-disabled awardees of disability insurance have significantly higher assets (87,017 dollars) than disabled recipients (73,911 dollars).

[5] We do not imply that current asset-tested programs are optimal.

Definition 1 *Given a tax system* $\{\tau_t\}$*, allocations of consumption, labor supply, and savings* $(\tilde{c}, \tilde{l}, \tilde{x}, \tilde{k})$ *constitute a competitive equilibrium if they solve the following problem:*

$$\max_{(c,l,x)\geq 0,\ k} \sum_{t=1}^{t=T} \beta^{t-1}\Pi_t \left[u\left(c_t\right) + v\left(l_t\right)\right] + \sum_{s=1}^{T} \Pi_{s-1}\left(1-\pi_s\right) \sum_{t=s}^{T} \beta^{t-1} u\left(x_t^s\right)$$

subject to

$$c_t + k_t \leq w\theta_t l_t + (1+R)k_{t-1} + \tau_t\left(\{\theta_i l_i, k_{i-1}\}_{i=1}^{i=t}\right) \text{ for } \forall t,$$

$$\begin{aligned} x_t^s + k_t^s &\leq (1+R)k_{t-1}^s + \tau_t \left(\begin{array}{c} \left(\{\theta_i l_i\}_{i=1}^{i=s-1}, \{\theta_i l_i = 0\}_{i=s}^{T}\right), \\ \left(\{k_{i-1}\}_{i=1}^{i=s}, \{k_i^s\}_{i=s}^{i=T}\right) \end{array} \right) \text{ for } t \geq s, \\ \text{where } k_{s-1}^s &= k_{s-1}, \end{aligned}$$

and feasibility (1) is satisfied.

We say that a tax system $\{\tau_t\}$ *implements* the optimal allocation (c^*, l^*, x^*) if the optimal allocation is equal to the competitive equilibrium allocation $(\tilde{c}, \tilde{l}, \tilde{x})$ defined above.

4.1 Linear savings tax does not implement the optimum

In this section, we present a two-period example that shows that a linear savings tax does not implement the optimum. We consider a setup in which agents live for two periods and are able in the first period of their lives, which is a special case of the more general model with $T = 2$ and $\pi_1 = 1$. When an agent is able, he has a skill $\theta = 1$. In the second period of his life, an agent is able with probability π and disabled with probability $(1-\pi)$. Denote first and second-period consumption of an able agent by c_1 and c_2, and second-period consumption of a disabled agent by x. Denote allocations of labor of able agents in periods 1 and 2 by l_1 and l_2, respectively. We define the optimal allocation $(c^*, l^*, x^*) = \{(c_1^*, c_2^*, x^*),\ (l_1^*, l_2^*)\}$.

One can conjecture (as in Diamond and Mirrlees 1978) that a linear savings tax that is equal to the intertemporal wedge in equation (3) and correctly chosen lump-sum taxes implement the optimal allocation. We show that this conjecture is false, as there exists a profitable deviation for an agent.

Consider a tax system that consists of a savings tax τ, a tax T_1 in period 1, and taxes T_a if an agent provides a positive amount of effective labor, and T_d if an agent does not work in period 2.

We now show that this system of taxes does not implement the optimal allocation.

Proposition 2 *The optimal allocation cannot be implemented with any tax system that uses only a linear tax on savings.*

Proof. Suppose the contrary. Then the savings tax must satisfy:

$$\tau = 1 - \left(\frac{u'(c_1^*)}{\beta[\pi u'(c_2^*) + (1-\pi)u'(x^*)]} - 1 \right) / R. \tag{4}$$

The lump sum taxes T_1, T_a, and T_d must satisfy:

$$c_1^* + k^* = wl_1^* + T_1, \tag{5}$$

$$c_2^* = (1 + R(1-\tau))k^* + wl_2^* + T_a, \tag{6}$$

$$x^* = (1 + R(1-\tau))k^* + T_d, \tag{7}$$

for some level of capital k^*.

An agent planning to claim disability in the second period, regardless of his true type, solves *Problem 1:*

$$\max_{(c,l,k)} u(c_1) + v(l_1) + \beta u(c_2)$$

subject to

$$c_1 + k_2 = wl_1 + T_1,$$

$$c_2 = (1 + R(1-\tau))k + T_d.$$

First note that (c_1^*, x^*, l_1^*), the allocation of a disabled agent under the optimum, is feasible for this problem. It is not a solution, however. To see this, notice that the first order necessary condition fails:

$$\begin{aligned} u'(c_1^*) &= (1 + (1-\tau)R)\beta[\pi u'(c_2^*) + (1-\pi)\, u'(x^*)] \\ &< (1 + (1-\tau)R)\beta u'(x^*). \end{aligned}$$

Hence, the maximized utility in Problem 1 exceeds the (ex post) realized utility, under the optimum, of an agent who is disabled in period 2.

Then notice that because the incentive constraint binds in the problem defining the optimal allocation, the agent's (ex ante) expected utility under that allocation is the same as his (ex post) realized utility conditional on being disabled:

$$u(c_1^*) + v(l_1^*) + \beta u(x^*) = u(c_1^*) + v(l_1^*) + \beta[\pi(u(c_2^*) + v(l_2^*)) + (1-\pi)u(x^*)].$$

Hence the maximized value in Problem 1 exceeds the ex ante expected utility under the optimal allocation. Analogous proof would hold for the case of arbitrary number of periods. ■

Intuitively, a linear savings tax is not sufficient to implement the optimal allocation because it is designed to preclude single deviations. Given that an agent tells the truth, a linear savings tax guarantees that he chooses the correct amount of savings. Given that an agent chooses a correct amount of savings, an agent chooses to tell the truth. Above we have shown that a joint deviation may be profitable in which an agent decides to jointly lie and change the amount of savings.[6]

We illustrate this intuition graphically. An agent who plans to claim disability in period 2 has utility $u(c_1) + \beta u(x)$. In Figure 1, we plot indifference curves for such an agent. By the incentive compatibility constraint, the utility of claiming disability in the social planner's problem, $u(c_1^*) + \beta u(x^*)$, is equal to the utility of telling the truth, and, therefore, is the utility of the optimal solution. Point A represents this choice of (c_1^*, x^*). In a competitive equilibrium with a linear tax τ, an agent's budget line, represented by the dashed line, has a slope of $-(1+R(1-\tau))$. Note that the slope of the indifference curve at point A is $-u'(c_1^*)/\beta u'(x^*)$. Therefore, the budget line intersects the indifference curve, and a point better than point A can be found by the agent. One can also see how asset testing would work: if a budget set represented by a solid line is constructed such that the indiffence curve only touches the budget set at point A, then point A would be chosen.

4.2 Asset-tested disability system implements the optimal solution

Let's formally define what comprises an asset-tested disability insurance program.

Definition 2 *An asset-tested disability insurance system* $(\bar{k}, S, T_a)$ *consists of:*

1. *a sequence of asset tests* $\bar{k}(i)$, $i = 1, ..., T$,

[6] A similar result is also derived in Albanesi and Sleet (2004) in their environment.

2. *a sequence of lump sum taxes of the form* $S_d(t,i) = T_d(i) - w\theta_t l_t$, $1 \leq i \leq t \leq T$, *where* $S_d(t,i)$ *is the transfer received in period* t *by a consumer who became newly disabled in period* i *and at that time had assets not exceeding* $\bar{k}(i)$,

3. *a lump sum tax* T_a *is paid each period by a consumer who is still working or who has assets exceeding* $\bar{k}(i)$ *when he declared disability.*

The theorem that follows states the main theoretical result of the paper: how to construct an asset tested disability system that implements the optimum.

Theorem 1 *For any constrained optimal allocation* (c^*, l^*, x^*), *there exists an asset-tested disability insurance program* $(\bar{k}, S, T_a)$ *for which* (c^*, l^*, x^*) *is a competitive equilibrium.*

Proof. See the appendix. ■

The logic behind the result is as follows. Consider an able agent of age t who plans to work when able and claim disability when disabled in period $t+1$. In period $t+1$, he receives income from savings and, in addition, income from working (if he remains able) or from disability transfers (if he becomes disabled). If instead the agent were to claim disability in period $t+1$ even if able, he would receive disability transfers instead of income from working. If those transfers are less than the income from working, the agent who contemplates falsely claiming disability has an incentive in period t to accumulate higher assets than he would if he planned to behave honestly. An asset test deters false claims by penalizing the strategy of oversaving and not working.

In Figure 1, we illustrate the intuition behind asset testing on a two-period example considered above. For $c_1 > c_1^*$ $(k > \bar{k})$, asset testing shifts the budget line down. The budget line is now represented by a solid line and has a discontinuity at point A. An agent who plans to claim disability in period 2, therefore, chooses point A. Point A gives the same utility as the utility of telling the truth under the optimal allocation. Therefore, an agent chooses the optimal allocation under the asset-tested disability system.

5 Quantitative Results

We first describe how we determine parameters of the model. We proceed to characterize a solution to the social planner's problem. We then evaluate the welfare benefits that a system with asset testing has over the best system without asset testing.

5.1 Parameterization

We choose the probability of becoming disabled using the data from McNeil (1997) who reports the number of self-reported disabled people by age groups[7]. We then calculate a conditional probability of becoming disabled and extrapolate the data to one-year intervals by fitting an exponential function. Table 1 reports shares of disabled people in our model by various age groups. We assume that 4 percent of the population is disabled at age 25, before entering the labor force. We compare the numbers we calculated to those reported in the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS). The SIPP estimates the number of people with severe disabilities, and the CPS reports the number of people with work disabilities. The CPS does not have information about work disabilities of people who are over age 65.

Table 1

Share of disabled population

Age groups	25-34	35-44	45-54	55-64	65-74
Model	5.2%	8.33%	13.97%	24.54%	43.19%
CPS (Stoddard et al. (1998))	5.5%	9.1%	13.2%	23.1%	n.a.
SIPP (McNeil (1997))	8.1%		13.9%	24.2%	30.7%

We assume that a period in the economy is one year and that each agent begins life at age 25 and lives to the age of 75 years. The utility function is chosen to be $ln(c) + aln(1 - l)$, where $a = 1.5$ is relative disutility of labor. We set the interest rate to match an annual interest rate net of depreciation R to be equal to 4.3 percent. Then we choose the discount factor $\beta = 0.958$ as a solution to $\frac{1}{\beta} = 1 + R$. We assume that an aggregate production function is a Cobb-Douglas production function with constant returns to scale $F(K, Y) = K^{\alpha}Y^{1-\alpha}$. In the production function, we set the capital share equal to $\alpha = 0.33$ and use the interest rate and the capital share to calculate the wage rate $w = 1.2243$. We use a lifetime skill profile that we obtained by fitting a quadratic function in the data from Rios-Rull (1996). Agents achieve the highest skill level at age 50 when

[7]These numbers need not reflect the true disability of these individuals. We follow Benitez-Silva et al. (2000) to argue that agents report their true status. Disability applicants may have a strong incentive to misreport their disability status to SSA but there is significantly weaker reason for respondents to misrepresent their information in *anonymous surveys* as any information they reported could not have any impact on the status of their disability benefits. One indication of respondents' truthfullness is provided by the fact that nearly 20% of disability recipients reported that they do not have a health problem that prevents them from working, and 5% of these recipients reported labor earnings in excess of the $500 per-month limit imposed by the SSA. Either of these self-reports constitute prima facie evidence for termination of benefits.

they are 45 percent more productive then at age 25. After age 50, skills decline and reach a minimum at age 75, roughly equal to the skill level at age 25.

5.2 Optimal system and implementation

In this subsection we numerically characterize an optimal disability insurance system and its implementation for the parametrized economy described above. We acknowledge that there are various reasons for retirement that are outside of the scope of this model. As the focus of the paper is on disability insurance, we force agents to retire at age 64 by setting $\pi_{40} = 1$. We then choose benefits for ages 64-75 optimally as they affect the dynamic incentives to claim disability at ages prior to their retirement.

We report optimal consumption profiles in panel A of the Figure 2. The upper solid line represents consumption c_t of agents who were able all their lives. This consumption is increasing with the duration of agent's work history as the social planner rewards the agent for working in period t by allocating him a higher continuation utility that leads to higher consumption at future dates.

The lower solid lline represents consumption x_t^t of a newly disabled agent. Note that we do not plot consumption x_s^t $(s > t)$ after an agent became disabled as it is constant and equal to x_t^t. The significant fall in consumption after an agent becomes disabled is necessary to ensure that able agents do not deviate and claim disability. There are two effects that determine consumption of the agent who becomes disabled. First, efficiency requires that more skilled agents work more, and, therefore, the consumption drop should be larger for such agents. We can see that agents who become disabled at ages 26 to 32 receive lower consumption then those who became disabled at the age of 25. The second effect comes from the intertemporal provision of the incentives. The planner rewards an agent for working by increasing the continuation utility when an agent becomes disabled. This effect calls for higher consumption for the agents who become disabled later in life and dominates the first effect when an agent reaches the age 32. The second effect increases with the work history, so that the consumption of disabled later in the lifecycle rises more steeply than for able agents.

A solid line in panel B of the Figure 2 represents optimal labor allocations l_t that are influenced by effects similar to the consumption profiles. On the one hand, it is optimal to require more productive agents to work more, and labor supply inherits the hump-shaped form of the skills profile. Agents who are 40 to 55 years old and have the highest skills spend about 45-50 percent of their time working. Younger and older people are not as productive and work less.On the other hand, intertemporal provision of incentives calls for an increase of the continuation utility of an able agent, which can be partially achieved by reducing amount of labor.[8]

An important feature of the model is the intertemporal distortion which we define as:

$$D_t = 1 - \left(\frac{u'(c_t^*)}{\beta[\pi_{t+1} u'(c_{t+1}^*) + (1 - \pi_{t+1})\, u'(x_{t+1}^{t+1*})]} - 1 \right) / R.$$

The intertemporal distortion depends on three factors: the probability of becoming disabled, the skill profile, and the length of work history. The probability of becoming disabled increases for older agents, thus making their future consumption more unpredictable, which increases the distortion. For higher-skilled agents the incentive problems are more severe, and they face a higher intertemporal wedge. The third factor, the length of work history, decreases the wedge. Agents with longer work history provide less labor and have a smaller variance of consumption. We find the intertemporal distortion to be quantitatively significant. The wedge grows from slightly below 3 percent at age 24 to 7 percent at age 50, and decreases almost to zero by age 63.

From the proof of Theorem 1 we calculate and plot transfers to disabled with the solid line in panel C of the Figure 2 and asset limits in panel D of the Figure 2. Note that we only plot disability transfers for newly disabled agents. Transfers are constant after an agent becomes disabled: for example, an agent who stops working at age 40 receives approximately 0.35 units of consumption for the rest of his life. Asset limits eventually increase because agents become wealthier as they accumulate more capital. That is also the reason that disability transfers eventually decrease, as agents receive a larger proportion of their income from savings. One interpretation of this system is that individuals who became disabled early in life receive large transfers, while those who become disabled later are supposed to supplement their lower disability transfers with savings accumulated while able.

5.3 Welfare benefits of asset testing

In this subsection we numerically compare the welfare of the best program without asset testing with that of the optimal insurance system.

[8] We also computed the optimal system for the case when the skills of the able are the same for all ages. In that case there is only the second effect, and the consumption of able and disabled agents monotonically increases with the length of work history, as there is no reason to require middle-aged agents to work more. The labor supply in this model monotonically decreases with work history (see Golosov and Tsyvinski 2005).

The optimal disability system without asset tests is a solution to the social planner's problem with hidden savings, an example of which is Diamond and Mirrlees (1995). Absence of asset testing implies that the planner does not have an ability to distort an intertemporal margin. The model with hidden savings is also similar to that of Werning (2001) and Abraham and Pavoni (2003). However, the dynamic first-order approach in these papers of imposing the Euler equation on the social planner's problem is invalid in our setup[9]. Our computational method for the model of hidden savings is similar to that in Golosov and Tsyvinski (2004). For each lifetime allocation of consumption and labor that a planner offers to an agent, we compute T optimal joint deviations in which an agent claims disability and chooses the optimal level of hidden savings, and an additional deviation in which an agent tells the truth but chooses a different level of savings than that prescribed by the planner. This method allows us to find a globally optimal solution to the social planner's problem with hidden savings.

In Figure 2, we plot with dashed lines the solution to the model without asset testing. The consumption profiles of a young able agent for the case of no asset testing starts below consumption of an able young for the case of asset testing and increases more rapidly. This rapid increase is the first source of the welfare loss compared to the optimal system, as agents prefer smooth consumption profiles. The decline in consumption for disabled agents who are 30-40 years old is large for the case of no asset testing. This decline is needed to ensure that an agent does not claim disability before becoming most productive. In the absence of asset testing, the planner has to penalize agents who declare disability early by giving them lower consumption than when asset testing is available, leading to the second source of welfare losses. A third source of welfare loss in the case of no asset testing is that the consumption profile of the disabled is less smooth than when asset testing is available. In particular, consumption allocations of disabled rise steeply after age 36. Labor profiles for both cases are virtually identical until about age 40. After age 40, the absence of asset testing implies that it is more difficult to provide incentives to work, and there is a smaller amount of labor provided. This leads to another source of welfare loss: a lower amount of labor implies a smaller amount of resources produced and available for redistribution to the disabled. It is evident from panel C that asset testing allows a significant increase in the level of disability transfers at most ages.

We find that a proportional increase in consumption by 0.5 percent for each history under the optimal system without asset testing produces the same lifetime utility as the lifetime utility in an

[9] See also Kocherlakota (2004a) for a discussion of the first order approach.

optimal system with asset testing. This number provides a lower bound on the welfare gains of switching to an optimal system as it represents gains solely of asset testing.

5.4 Robustness of quantitative results

We also considered a model of Social Security as optimal disability insurance. One of the explanations for the existence of the Social Security system is its role as an optimal "retirement insurance". Diamond and Mirrlees (1978, pp. 331-332) view a setup similar to ours as a general way of modelling the Social Security system, including the old age portion (also see Mulligan and Sala-i-Martin 1999). The Social Security system can be viewed as mandatory government insurance against becoming disabled (not being able to work) at old ages. While Social Security benefits are conditioned on retirement, in this modification of the model we condition benefits on a more fundamental risk, disability.

In this model, agents live for 75 years, the probability of becoming disabled is computed to this age, and there is no mandatory requirement. In fact, all agents who are able at ages 65-75 work in the example that we compute. However, the model features endogenous retirement, as older agents tend to work significantly less than young agents with similar skills. Agents who are 75 year old and still productive spend less than 10 percent of their time working. We find that welfare gains of asset-testing increase modestly to 0.65 percent of consumption, as agents, even without mandatory retirement, of age 65-75 provide a low amount of labor.

We already discussed a modification of the model in which the skill profile for the able is constant over lifetime. The welfare benefits of asset testing in this model are equal to about 0.35 percent. We also calculated a model in which the probability of disability is half of that used in this paper. The welfare gain of asset testing is similar to the one derived in the benchmark model and is equal to 0.3 percent, as the size of the informational friction decreases with the smaller probability of disability.

We also calculated a stylized current social insurance system to compare with the optimal system described above. Since disability insurance is an integral part of the social insurance system, we model the current social insurance system as a joint disability and retirement system. An agent can stop working either because he is truly disabled or because social insurance transfers create a disincentive to work. If an agent does not work, he receives a social security transfer. An agent can save at a rate R that is taxed at the rate τ. When an agent stops working, he receives a disability transfer T_d independent of the age. In the supplement to this paper (Golosov and Tsyvinski 2005) we provide a detailed description of the stylized current system. The welfare gain of a switch to the optimal insurance system from a stylized current social security system is equivalent to an increase of consumption by 2.8 percent for each history. The welfare gain mainly comes from the increase in benefits to agents who became disabled relatively early in their lives.

6 Final remarks, robustness, and the role of assumptions

In this paper we consider the problem of implementation of optimal disability insurance when disability status is unobservable and show what instruments can implement the optimum. Asset testing allows to control joint deviations in which an agent, in anticipation of falsely claiming disability, increases her savings compared to those implied by the optimal allocation. Studying implementation is important, as the existing literature on optimal social insurance often stops at characterizing an optimal allocation without studying taxes that implement the optimum. We then provide numerical results that suggest that asset testing may be quantitatively important.

We made two assumptions that are important for characterizing implementation of the optimum. First, disability is an absorbing state. This assumption reduces the number of histories that we need to consider. We have to keep track only of an agent's age and the age when he claimed disability. An interesting extension would be to study an economy in which disability is not permanent but there is a small probability of recovery. In that case, optimal disability benefits also have to encourage individuals who recover from disability to leave disability rolls. If skills follow a more general process such as non-permanent disability, a taxation mechanism such as a modification of Albanesi and Sleet (2004) to the case of persistent shocks or a method of Kocherlakota (2004a) may be needed to implement the optimum. The second assumption that we make is that a disabled agent has zero skill. This assumption allows us not to consider deviations in which a disabled agent pretends to be able or more complicated deviations in which an agent undersaves and works too much. We conjecture that if the skill of a disabled agent is sufficiently close to zero the implementation that we derived still remains valid.

We also treated government as the only provider of disability insurance without considering insurance that is provided by private markets. This assumption seems to be close to reality. Except for SSDI, few other options provide protection against disability risk. For example, only 25 percent of private-sector employees receive long-term disability coverage (SSA 2001). In Golosov and Tsyvinski 2004 we showed that in the environment in which consumption is observable, publicly provided insurance is as efficient as insurance provided by private markets. In particular, if all insurance is provided by private intermediaries then insurance contracts would feature exactly the same asset-tested disability benefits as the ones described in this paper.

Theoretical results of the model are robust to two extensions. First, consider a case of observable heterogeneity. Suppose that there are i types of agents who *observably* differ in the probability of becoming disabled, discount factors, skill profiles when able, etc. It is easy to show using the same proof as in the paper that an asset test *conditional on type* implements the optimal allocation. Examples of insurance that is conditioned on agent's type such as gender abound in the practice of private insurance. We can also consider an environment in which there are multiple unobservable levels of skills that follow a general stochastic process but there exists an absorbing disability state in which disabled agents cannot work. Assume that allocations of consumption and effective labor for all histories except for disability states are provided by a direct mechanism. Then it is easy to show that an asset-tested disability insurance implements the optimum. Moreover, the asset test has to be conditioned on assets of a marginal agent. One interpretation of this model is a joint system of optimal taxation and disability insurance. Insurance for all skill shocks with the exception of disability is accomplished through a direct mechanism that stands in for the income and wealth tax system. Disability insurance is achieved through an asset-tested disability system.

The results in our model as well as in other models of optimal dynamic taxation are not robust to inclusion of unobservable heterogeneity such as, for example, differential unobserved discount rates. The main technical difficulty is that, even in the static model, the problem becomes the one of multidimensional screening. In the case of one unobservable characteristic it is easy to show that incentive compatibility constraints are binding from the high to the low types. The major difficulty with the multidimensional screening is determining a pattern of binding incentive constraints[10].

While the described extensions are interesting, the magnitude of the welfare gains of asset testing gives us confidence that the forces we have captured in this paper are significant from both the theoretical and policy-making perspectives.

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7 Appendix

7.1 Proof of Theorem 1.

The proof is by construction. Choose the tax on able, T_a, to satisfy[11]

$$\sum_{t=1}^{t=T} c_t^* \beta^{t-1} = \sum_{t=1}^{t=T} w\theta_t l_t^* \beta^{t-1} + T_a \sum_{t=1}^{t=T} \beta^{t-1}.$$

Let the transfers to disabled, $T_d(j)$, satisfy:

$$\sum_{t=1}^{t=j-1} c_t^* \beta^{t-1} + \sum_{t=j}^{t=T} x_t^{j*} \beta^{t-1} = \sum_{t=1}^{t=j-1} \left[w\theta_t l_t^* + T_a \right] \beta^{t-1} + T_d(j) \sum_{t=j}^{t=T} \beta^{t-1}.$$

Finally, set the asset limits $\bar{k}_t$ are defined recursively:

$$c_t^* + \bar{k}_{t+1} = w\theta_t l_t^* + \frac{\bar{k}_t}{\beta} + T_a$$

with $\bar{k}_1 = 0$.

Using this policy we can re-write the feasibility constraint:

$$\sum_{t=1}^{t=T} \beta^{t-1} \Pi_t T_a + \sum_{s=1}^{T} \Pi_{s-1} \left(1 - \pi_s\right) \sum_{t=s}^{T} \beta^{t-1} T_d\left(s\right) \leq 0. \tag{8}$$

First, we prove that the expected present value of transfers for the able agent is lower than the present value of disability transfers. That is $\{T_a, T_d(t)\}$, defined above, satisfy, for all $t = 1...T$,

$$\begin{aligned} & T_a + \sum_{i=t+1}^{i=T} \left[\Pi_{t+1,i-1} \left(1 - \pi_i\right) \left(\sum_{s=i}^{s=T} T_d(i) \beta^{s-t} \right) + \Pi_{t+1,i} T_a \beta^{i-t} \right] \\ \leq & \sum_{i=0}^{i=T-t} T_d(t) \beta^i. \end{aligned} \tag{9}$$

Note that (9) is equivalent to

[11] Note that agents who do not receive disability transfers face a tax T_a regardless of their age. Without loss of generality, we could have assumed that these taxes are indexed by age. In that case the levels of asset tests would not be uniquely pinned down.

$$\begin{aligned} & c_t^* - w\theta_t l_t^* + \sum_{i=t+1}^{i=T} \left[\Pi_{t+1,i-1} \left(1-\pi_i\right) \left(\sum_{s=i}^{s=T} x_s^{i*} \beta^{s-t} \right) + \Pi_{t+1,i} \left(c_i^* - w\theta_i l_i^*\right) \beta^{i-t} \right] \\ \leq & \sum_{i=0}^{i=T-t} x_{t+i}^{t*} \beta^i. \end{aligned} \tag{10}$$

Suppose equation (10) did not hold for some t. Then the social planner could give consumption of the disabled $\{x_{t+i}^{t*}\}_{i=0}^{i=T-t}$ to the agents who are still able in period t and set their labor to zero. Since the period t incentive constrain holds with equality, the utility of the agent does not change. The new allocation is still incentive compatible, but the feasibility constraint is relaxed. The social planner can further improve upon such an allocation, therefore, (c^*, l^*, x^*) cannot be an optimum.

Next we will show that (9) implies $T_a \leq T_d(t)$ for all t.

For $t = T$ this fact is immediate from (9). For $t = T-1$, (9) says:

$$\begin{aligned} & T_a + \beta(\pi_T T_a + (1-\pi_T)\, T_d(T)) \\ \leq \;& T_d(T-1)\,(1+\beta)\,. \end{aligned}$$

Since $T_a \leq T_d(T)$ the above equation implies $T_a \leq T_d(T-1)$. Continue by induction to establish the claim for all t.

Consider the asset-tested system constructed as described above. Pick any allocation $(\tilde{c}, \tilde{l}, \tilde{x})$ and saving decisions $(\tilde{k})$ that maximizes an agent's utility. We will show that the utility from such allocations cannot be higher then the utility from (c^*, l^*, x^*).

Step 1: There exists a utility maximizing allocation $(\tilde{c}, \tilde{y}, \tilde{k})$ such that an agent never claims disability if he is able.

Suppose that an agent is strictly better off by claiming disability if he is able in some period j. The agent can claim disability in period j only if his assets in that period are $\tilde{k}_j \leq \bar{k}_j$. Suppose $\tilde{k}_j = \bar{k}_j$. By construction, the maximum utility the agent can obtain if his assets are $\bar{k}_j$ and his taxes are $T_d(j) - w\theta_t l_t$ for all the subsequent periods is $u(x_j^{j*}) + ... + \beta^{T-j} u(x_T^{j*})$, which is the utility that the planner allocates to the agent who becomes disabled in period j. But the agent with assets $\bar{k}_j$ in period j can choose the future path $(\{c^*, l^*, x^*\}_{t=j}^T)$, as it is in his budget constraint. By the incentive compatibility of the optimal allocations, this future path gives weakly higher utility than claiming disability in period j.

Alternatively, suppose $\tilde{k}_j < \bar{k}_j$. The agent's utility maximization implies that $\tilde{x}_j^j < x_j^{j*}$. The allocation is utility maximizing in this case if $u'(\tilde{c}_{j-1}) = u'(\tilde{c}_j^j)$. If this Euler equation did not hold, an agent could transfer a small amount of resources ε intertemporally. Such a transfer still allows him to claim disability in period j and gives strictly higher utility. Since $\tilde{x}_j^j < x_j^{j*}$ this implies together with Corollary 1 that $u'(\tilde{c}_{j-1}) > u'(c_{j-1}^*)$. The agent's budget constraint $\tilde{c}_{j-1} + \tilde{k}_j = w\theta_{j-1}\tilde{l}_{j-1} + \frac{1}{\beta}\tilde{k}_{j-1}$ and intratemporal optimality condition

$$-v'\left(\tilde{l}_{j-1}\right)\frac{1}{\theta_{j-1}} = u'\left(\tilde{c}_{j-1}\right)w,$$

imply that $\tilde{k}_{j-1} < \bar{k}_{j-1}$. We can continue backward to show that $\tilde{k}_t < \bar{k}_t$ for all $t < j$. However, this implies that $\tilde{k}_1 < \bar{k}_1 = 0$, which is impossible. We showed that there exists a utility maximizing allocation in which an agent never claims disability when he is able.

Step 2: The constructed asset-tested system implements the optimum.

We show that if the conditions of Step 1 are satisfied, the utility maximizing allocation must be feasible and incentive compatible. Therefore, it cannot give a higher utility than (c^*, l^*, x^*).

The allocation is incentive compatible since it comes from the agent's maximization problem.

From Step 1, the able agent always receives a transfer T_a. We showed that $T_a \leq T_d(t)$ for all t, so that this is the lowest possible transfer (the highest tax since $T_a \leq 0$) the agent can receive. Note that if an agent saves more than $\bar{k}_i$ in some period $i-1$ and becomes disabled in period i, he receives transfer T_a until his savings fall below the asset limit, after which he is entitled to $T_d(i)$. The present value of such transfers is lower than the present value of the transfers to the agent who could claim disability in period i, which is equal to $T_d(i)\left(1 + ... + \beta^{T-i}\right)$.

Therefore, the ex ante expected value of transfers cannot be higher than

$$\sum_{t=1}^{t=T}\left(\left(\sum_{i=1}^{i=t}\Pi_{i-1}\left(1-\pi_i\right)T_d(i)\beta^{t-1}\right) + \Pi_t T_a \beta^{t-1}\right) \leq 0$$

and, from (8), the allocation that has such transfers must be feasible.

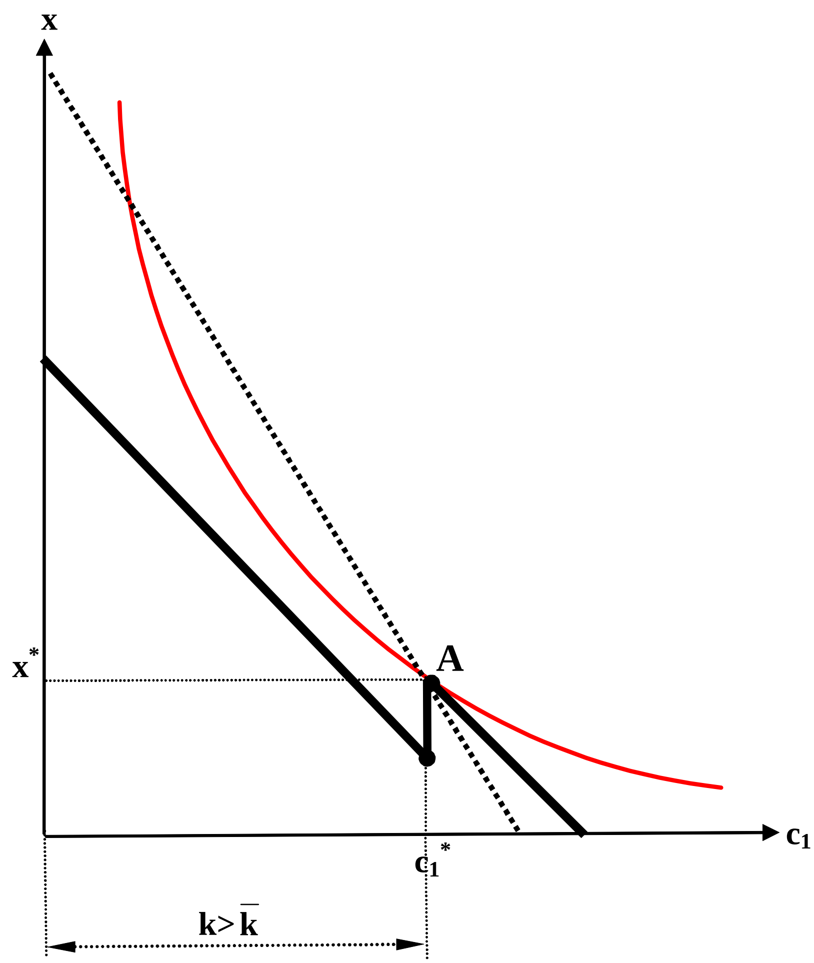

Figure 1:

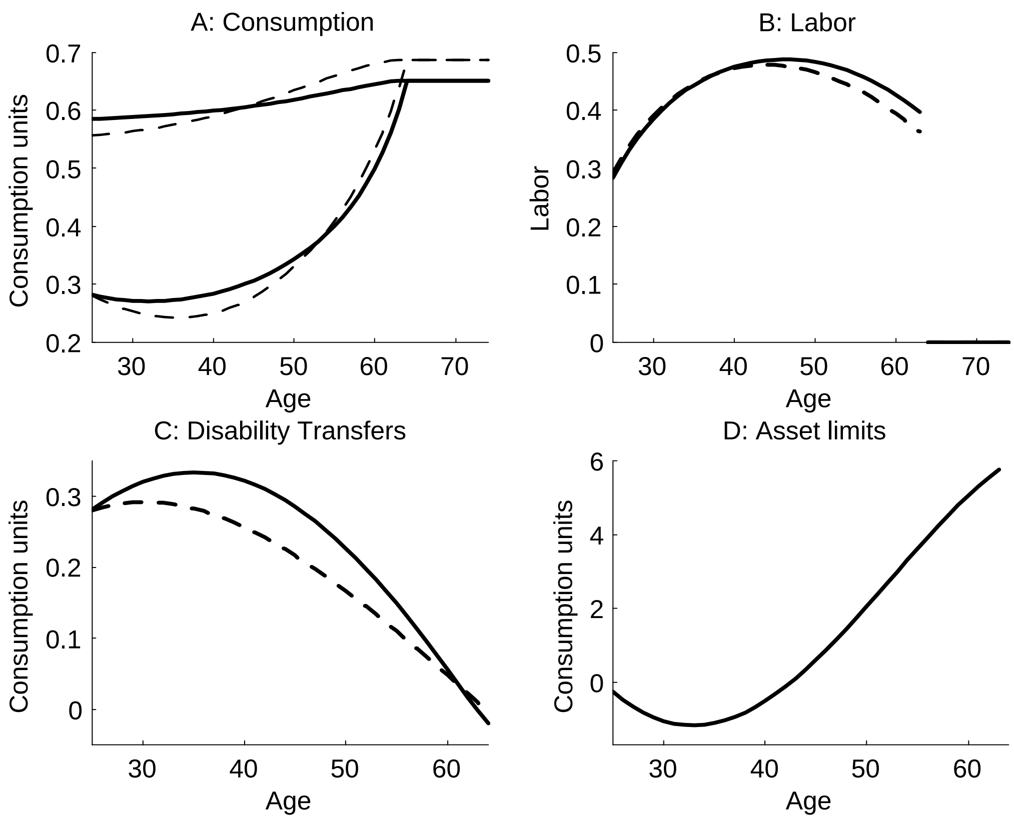
A: Consumption
Consumption units
0.7
0.6
0.5
0.4
0.3
0.2
30
40
50
60
70
Age
B: Labor
Labor
0.5
0.4
0.3
0.2
0.1
0
30
40
50
60
70
Age
C: Disability Transfers
Consumption units
0.3
0.2
0.1
0
30
40
50
60
Age
D: Asset limits
Consumption units
6
4
2
0
30
40
50
60
Age

Figure 2: