

# **Labor-Augmenting and Skill-Biased Technical Change**

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**based in part on joint work with Veronica Guerrieri**

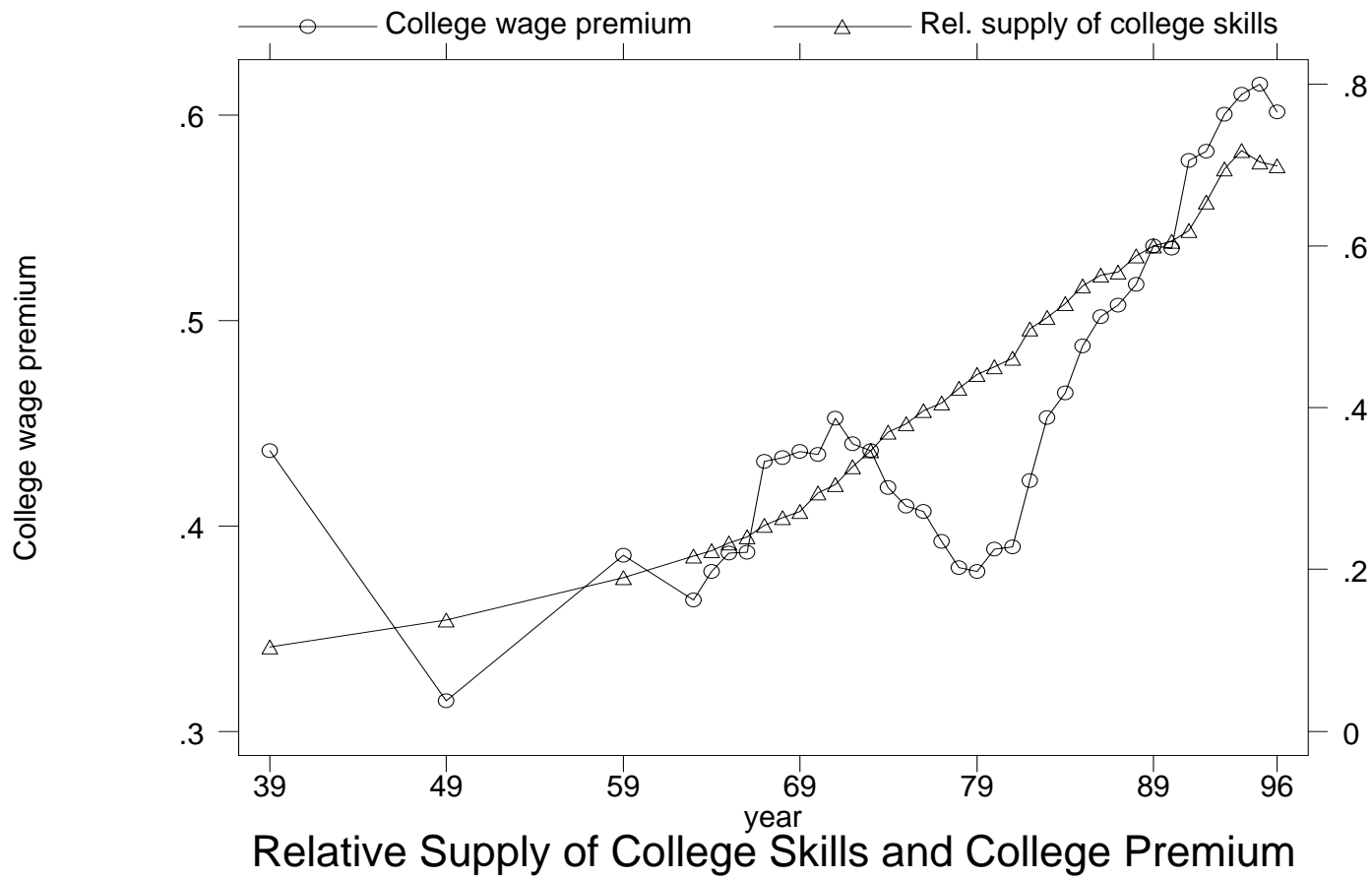
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## **Introduction**

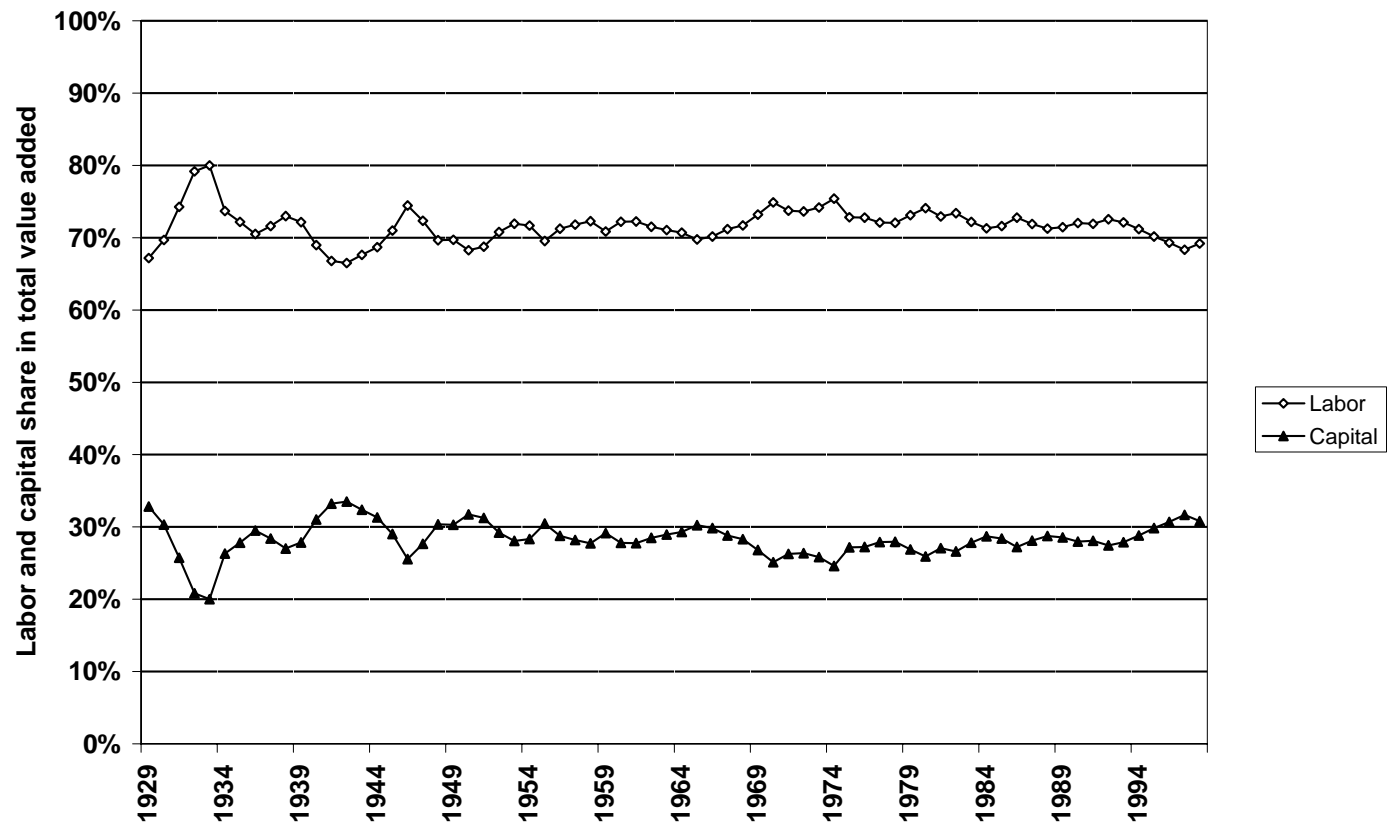
- Technical change is often not neutral towards different factors of production.
- Theoretical question: what determines the bias and the direction of technical change?
- This talk synthesizes some recent work on models of “directed technical change” and offers some new models.

## The Form of Technical Change

- Notable features of 20th-century U.S. technology:
  1. Skill-biased technical change throughout the 20th century, and possible acceleration in skill-biased technical change during the past 25 years (Figure 1).
    - But 19th-century technical change most likely skill replacing.
  2. Capital share in GDP constant despite significant capital deepening, most likely due to the fact that long run technical change purely labor augmenting (Harrod neutral).  
Figure 2.
- Question: What explains these various biases?



Relative Supply of College Equivalent Workers and the College Premium in the U.S., 1949-1995.



Capital and Labor Shares in the U.S., 1929-1999.

## **Framework (Acemoglu, QJE 1998, RES 2002)**

- Profit incentives determine what type of technologies are developed.
- When developing technologies complementing a particular factor (say skilled workers) is more profitable, more of these technologies will be developed.
- Key role of the elasticity of substitution between factors.

## Main Forces

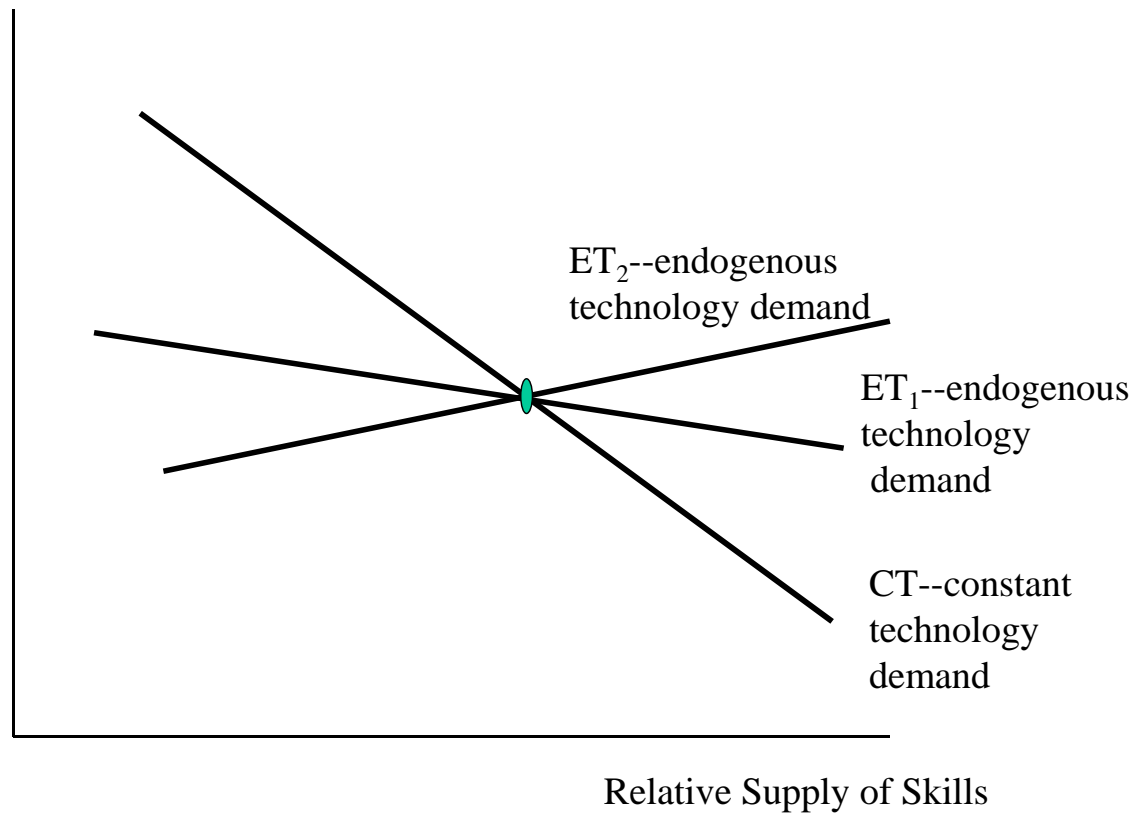
- What determines the relative profitability of developing different technologies?
  1. The price effect: there will be stronger incentives to develop technologies when the goods produced by these technologies command higher prices.
  2. The market size effect: it is more profitable to develop technologies that have a larger market.

Schmookler (1966): “invention is largely an economic activity which, like other economic activities, is pursued for gain;... expected gain varies with expected sales of goods embodying the invention.”
- The elasticity of substitution between the factors determines the relative strengths of these two effects.

## Major Results

- For simplicity, suppose there are two factors of production.
- Two main propositions:
- **Weak Endogenous Bias:** As long as the elasticity of substitution between factors not equal to 1, then an increase in the supply of a factor causes technology to become endogenously biased towards that factor.
- **Weak Endogenous Bias (alternative form):** As long as the elasticity of substitution between factors not equal to 1, then the endogenous-technology factor demand curves are more “elastic” than constant-technology demand curves.
- Similarity to LeChatelier Principle.

Skill premium



Relative demand for skills.

## Major Results (Continued)

- **Weak Endogenous Bias (relation to augmenting change):** If the elasticity of substitution between factors is greater than 1, then an increase in the supply of a factor causes technology to become endogenously “augmenting” that factor. If the elasticity of substitution is less than 1, an increase in the supply of factor causes technology to become endogenously “augmenting” the other factor.
- **Strong Endogenous Bias:** If the elasticity of substitution between factors sufficiently greater than 1, then the endogenous-technology demand curves will be upwards sloping.

## Implications

- Secular skill biased technical change because of increased educational attainment.
- Consistent with acceleration in skill bias in the face of acceleration college graduation during the 1970s.
- Consistent with skill-replacing technical change during the 19th-century when the supply of unskilled labor increased substantially.
- Provided that elasticity of substitution between labor and capital less than 1, consistent with primarily labor-augmenting technical change because of capital deepening.

## Problems and Current Research

- Yet technical change primarily labor-augmenting, but not purely labor augmenting (not Harrod neutral) unless much more structure imposed on the innovation possibilities frontier (Acemoglu, JEEA 2003).
- No natural framework where skill-biased and labor-augmenting technical change can be analyzed jointly.
- Growth always “balanced”, while in the data, aggregate growth satisfies the Kaldor facts, but also major sectoral changes.
- New research (Acemoglu and Guerrieri, Non-Balanced Endogenous Growth): towards a framework for analysis of non-balanced growth path with endogenous technology (both skill biased and endogenously labor augmenting).

## Results in Acemoglu and Guerrieri

- Asymptotic non-balanced growth equilibrium with the interest rate and the share of capital in GDP constant.
  - Different sectors grow at different rates.
  - High-skill services slower real faster nominal growth than the rest of the economy.
- Short-run elasticity of substitution between capital and labor inputs  $< 1$ .
- Capital-skill complementarity;  $K \uparrow \implies$  the skill premium  $\omega \uparrow$
- Skill-biased technical change in the long run (even if  $H$  grows at the same rate as  $L$ )  $\implies \dot{\omega} > 0$  in the long run.
- Technical change typically “capital-augmenting”; only in the limit Harrod neutral.

## Earlier Literature

- Hicks was the first to discuss determinants of biased technical change (but also Marx)

“A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive...

The general tendency to a more rapid increase in capital than labor which has marked European history during the last few centuries has naturally provided a stimulus to labor-saving invention” (pp. 124-5).

- Kennedy: innovation possibilities frontier determines factor distribution of income.

## Relation to Earlier Literature

- This earlier literature criticized because of lack of microfoundations: increasing returns at the firm level when firms choose technology, arbitrary assumptions, etc..
- The approach here builds on endogenous technical change models to generate a well-defined decentralized equilibrium (e.g., Romer, Lucas, Grossman and Helpman, Aghion and Howitt, Stokey, Young).

## Factor-augmenting and Factor-biased Change

- Consider the constant elasticity of substitution (CES) production function

$$y = \left[ \gamma (A_L L)^{\frac{\sigma-1}{\sigma}} + (1 - \gamma) (A_Z Z)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

- Here  $\sigma \in (0, \infty)$  is the elasticity of substitution between the two factors.
- $A_L$  is labor-augmenting (labor-complementary) and  $A_Z$  is Z-complementary.

## Factor-augmenting and Factor-biased Change (continued)

- The relative marginal product of the two factors:

$$\frac{MP_Z}{MP_L} = \frac{1 - \gamma}{\gamma} \left( \frac{A_Z}{A_L} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{Z}{L} \right)^{-\frac{1}{\sigma}}. \quad (1)$$

- This implies that when  $\sigma > 1$ , i.e., when the two factors are gross substitutes,  $A_L$  is labor-biased and  $A_Z$  is Z-biased.
- When  $\sigma < 1$ , i.e., when the two factors are gross complements,  $A_Z$  is labor-biased and  $A_L$  is Z-biased.
- Important contrast between physical products and marginal products.

## The Environment

- Preferences

$$\int_0^{\infty} \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad (2)$$

- The budget constraint:

$$C + I + R \leq Y \equiv \left[ \gamma Y_L^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_Z^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (3)$$

- Output aggregate produced from two other (intermediate) goods,  $Y_L$  and  $Y_Z$ , with elasticity of substitution  $\epsilon$ .

## Intermediate Good Production Functions

- Production functions:

$$Y_L = \frac{1}{1-\beta} \left( \int_0^{N_L} x_L(j)^{1-\beta} dj \right) L^\beta, \quad (4)$$

and

$$Y_Z = \frac{1}{1-\beta} \left( \int_0^{N_Z} x_Z(j)^{1-\beta} dj \right) Z^\beta, \quad (5)$$

## Market Structure

- Assume that machines to both sectors are supplied by “technology monopolists” and take the state variables  $N_L$  and  $N_Z$  as given.
- Each monopolist sets a rental price  $\chi_L(j)$  or  $\chi_Z(j)$  for the machine it supplies to the market.
- The marginal cost of production is the same for all machines and equal to  $\psi \equiv 1 - \beta$  in terms of the final good (normalization without loss of generality).

## Definition of Equilibrium

- An equilibrium in this economy is defined by dynamic paths for:
  - factor, intermediates and final goods prices  $r, w_L, w_H, [\chi_H(i)]_{i=1}^{N_H}, [\chi_L(i)]_{i=1}^{N_L}, p_H$  and  $p_L$ , that ensure market clearing
  - consumption decisions that maximize the utility of the representative agent
  - employment and research decisions that maximize firm profits

## Equilibrium

- Price taking implies

$$\max_{L, \{x_L(j)\}} p_L Y_L - w_L L - \int_0^{N_L} \chi_L(j) x_L(j) dj, \quad (6)$$

- This gives machine demands as

$$x_L(j) = \left[ \frac{p_L}{\chi_L(j)} \right]^{1/\beta} L. \quad (7)$$

- Similarly

$$x_Z(j) = \left[ \frac{p_Z}{\chi_Z(j)} \right]^{1/\beta} Z, \quad (8)$$

## Equilibrium (continued)

- Since the demand curve for machines facing the monopolist, (7), is iso-elastic, the profit-maximizing price will be a constant markup over marginal cost.
- So all machine prices will be

$$\chi_L(j) = \chi_Z(j) = 1.$$

- Profits of technology monopolists are obtained as

$$\pi_L = \beta p_L^{1/\beta} L \text{ and } \pi_Z = \beta p_Z^{1/\beta} Z. \quad (9)$$

## Value Functions

- Let  $V_Z$  and  $V_L$  be the net present discounted values of new innovations. Then in steady state:

$$V_L = \frac{\beta p_L^{1/\beta} L}{r} \quad \text{and} \quad V_Z = \frac{\beta p_Z^{1/\beta} Z}{r}. \quad (10)$$

- The greater is  $V_Z$  relative to  $V_L$ , the greater are the incentives to develop  $Z$ -complementary machines,  $N_Z$ , rather than  $N_L$ .
  1. The price effect: a greater incentive to invent technologies producing more expensive goods.
  2. The market size effect: a larger market for the technology leads to more innovation. The market size effect encourages innovation for the more abundant factor.

## Value Functions (alternative form)

- Alternatively

$$V_L = \frac{(1 - \beta) w_L L}{r N_L} \text{ and } V_Z = \frac{(1 - \beta) w_Z Z}{r N_Z}. \quad (11)$$

- The two effects correspond to:
  - wage effect, and
  - market size effect.

## Importance of the Elasticity of Substitution

- Substituting for relative prices, relative profitability is

$$\frac{V_Z}{V_L} = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{\epsilon}{\sigma}} \left( \frac{N_Z}{N_L} \right)^{-\frac{1}{\sigma}} \left( \frac{Z}{L} \right)^{\frac{\sigma - 1}{\sigma}}. \quad (12)$$

where

$$\sigma \equiv \epsilon - (\epsilon - 1)(1 - \beta).$$

is *derived* the elasticity of substitution between the two factors.

- An increase in the relative factor supply,  $Z/L$ , will increase  $V_Z/V_L$  as long as  $\sigma > 1$  or equivalently  $\epsilon > 1$ , and it will reduce it if  $\sigma < 1$ .
- Therefore, the elasticity of substitution regulates whether the price effect dominates the market size effect.

## Basic Forces

- If the elasticity of substitution between factors,  $\sigma$  is greater than 1, then an increase in  $Z/L$  makes Z-augmenting technical change more profitable.
  - Therefore, all else equal, this should induce Z-augmenting technical change.
- If the elasticity of substitution between factors,  $\sigma$  is less than 1, then an increase in  $Z/L$  makes L-augmenting technical change more profitable.
  - Therefore, all else equal, this should induce L-augmenting technical change.

## Physical Products Versus Marginal Products

- Relative factor prices are

$$\frac{w_Z}{w_L} = p^{1/\beta} \frac{N_Z}{N_L} = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\epsilon}{\sigma}} \left( \frac{N_Z}{N_L} \right)^{-\frac{\sigma-1}{\sigma}} \left( \frac{Z}{L} \right)^{-\frac{1}{\sigma}}. \quad (13)$$

- First, the relative factor reward,  $w_Z/w_L$ , is decreasing in the relative factor supply,  $Z/L$ .

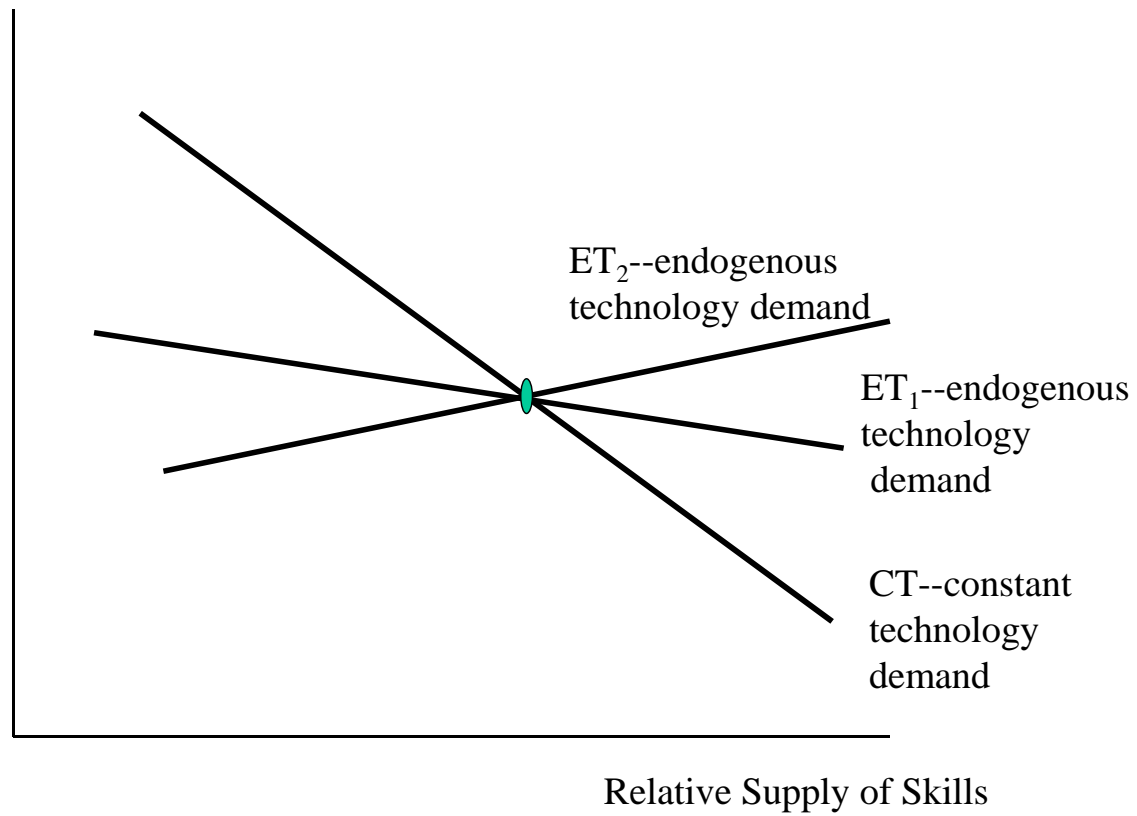
## Physical Products Versus Marginal Products (continued)

- Second and more important, the same combination of parameters,  $\sigma \equiv \epsilon - (\epsilon - 1)(1 - \beta)$ , which determines whether innovation for more abundant factors is more profitable, also determines whether a greater  $N_Z/N_L$ —the relative physical productivity of factor Z—increases  $w_Z/w_L$
- When  $\sigma > 1$  (which in turn happens when  $\epsilon > 1$ ), greater  $N_Z/N_L$  increases  $w_Z/w_L$ , but when  $\sigma < 1$ , it has the opposite effect.
- Implication: our first major result.

## Towards The First Major Result

- As long as the elasticity of substitution between factors,  $\sigma$  is not equal to 1, then an increase in  $Z/L$  makes Z-biased technical change more profitable.
  - Therefore, all else equal, this should induce Z-biased technical change.

Skill premium



Relative demand for skills.

## Innovation Possibilities Frontier

- Suppose innovations are produced using final goods (“the lab equipment” specification of innovations)

$$\dot{N}_L = \eta_L R_L \text{ and } \dot{N}_Z = \eta_Z R_Z, \quad (14)$$

where  $R$  denotes R&D expenditure.

- From  $V_L = V_Z$ , this gives the following long-run “technology market clearing” condition:

$$\eta_L \pi_L = \eta_Z \pi_Z. \quad (15)$$

## Weak Endogenous Bias

- Then, relative physical productivities can be solved for

$$\frac{N_Z}{N_L} = \eta^\sigma \left( \frac{1 - \gamma}{\gamma} \right)^\epsilon \left( \frac{Z}{L} \right)^{\sigma-1}. \quad (16)$$

- Recall that measure of Z-biased technical change is

$$\left( \frac{N_Z}{N_L} \right)^{\frac{\sigma-1}{\sigma}} \quad (17)$$

**Result 1 (Weak Endogenous Bias):**  $(N_Z/N_L)^{\frac{\sigma-1}{\sigma}}$  is increasing in  $Z/L$  as long as  $\sigma$  is not equal to 1.

## Weak Endogenous Bias—Relation to Augmenting Technical Change

- Another immediate result is:

**Result 2 (Augmenting Technical Change):**  $N_Z/N_L$  is increasing in  $Z/L$  when  $\sigma > 1$  and is decreasing in  $Z/L$  when  $\sigma < 1$ .

- Intuitively: when the elasticity of substitution between factors is less than 1, Z-augmenting technical change becomes L-biased.

## Weak Endogenous Bias (Alternative Form)

- Relative factor rewards are

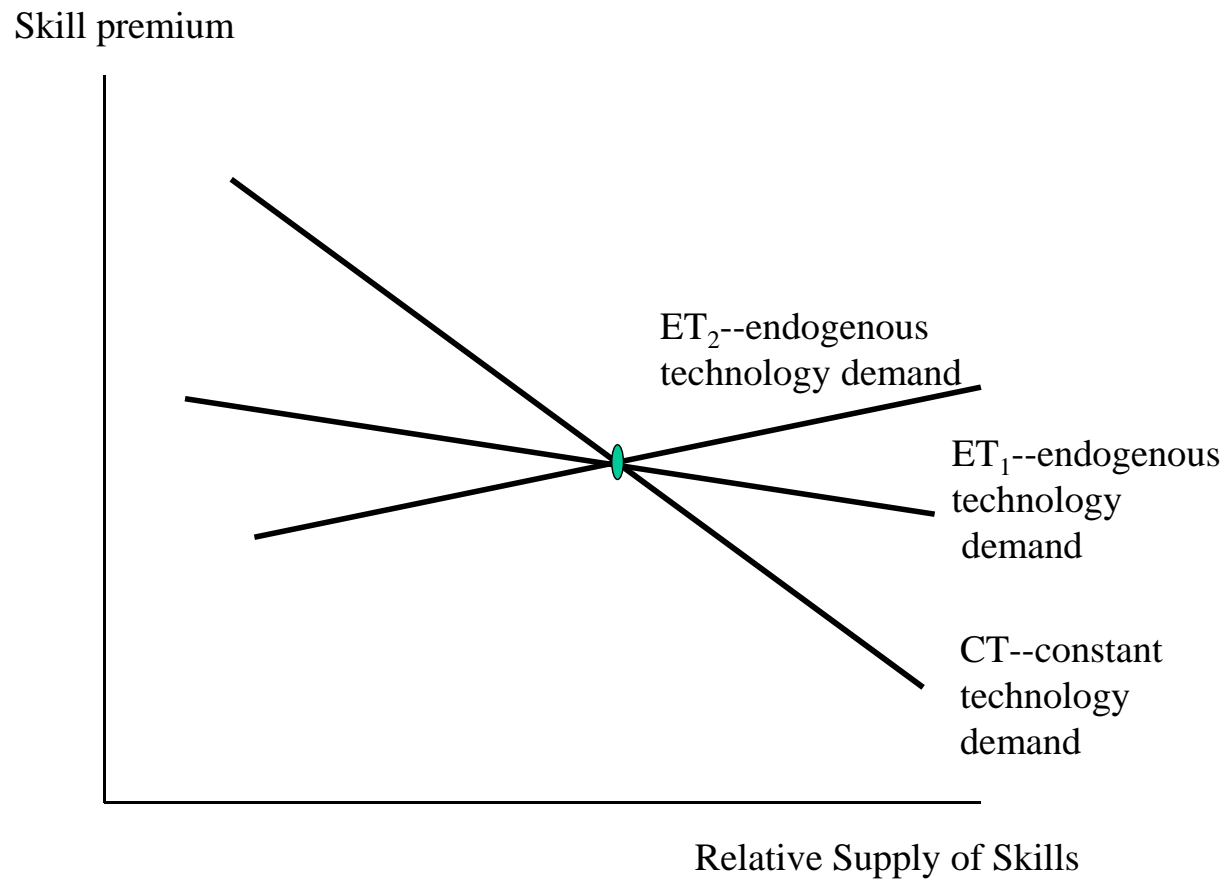
$$\frac{w_Z}{w_L} = \eta^{\sigma-1} \left( \frac{1-\gamma}{\gamma} \right)^\epsilon \left( \frac{Z}{L} \right)^{\sigma-2}. \quad (18)$$

- Comparing this equation to the relative demand given technology, we see that the response of relative factor rewards to changes in relative supply is always more elastic in (18).

### **Result 3 (Weak Endogenous Bias—Alternative Form):**

As long as  $\sigma$  is not equal to 1, the endogenous-technology factor demand curves are more elastic than the constant-technology demand curves.

- Application of the LeChatelier principle.



Relative demand for skills.

## Strong Endogenous Bias

- The more important and surprising result here is that if  $\alpha$  is sufficiently large, in particular if  $\sigma > 2$ , the relationship between relative factor supplies and relative factor rewards can be upward sloping.

**Result 4 (Strong Endogenous Bias):** If  $\sigma > 2$ , then  $w_Z/w_L$  is increasing in  $Z/L$ .

## Dynamics

- It can also be shown that this is the unique balanced growth path
- Moreover, this balanced growth path is globally stable.

## Endogenous Skill-Biased Technical Change

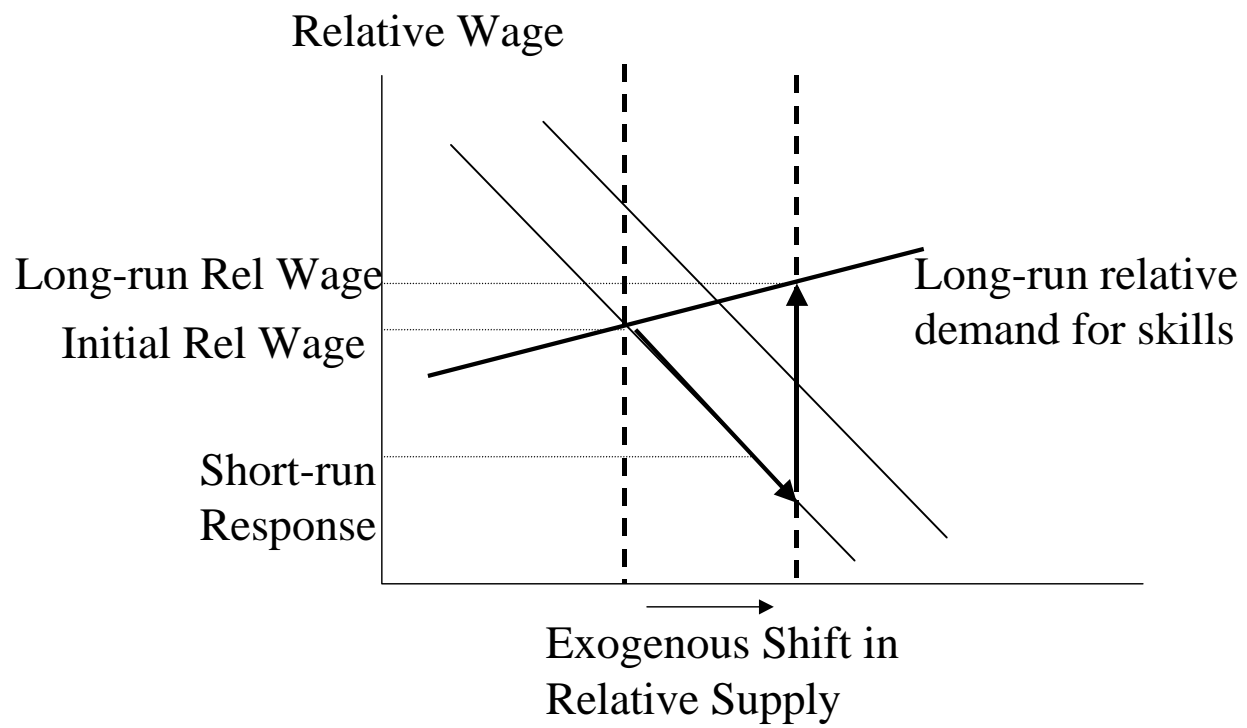
- Now consider an application where  $Z$  corresponds to skilled labor,  $H$ .
- The skill premium is given by

$$\text{skill premium} = \eta^\sigma \left( \frac{1-\gamma}{\gamma} \right)^\epsilon \left( \frac{H}{L} \right)^{\sigma-2}. \quad (19)$$

- If  $\sigma > 2$ , then the long-run relationship between the relative supply of skills and the skill premium is positive

## Implications

- Weak endogenous bias possible explanation for
  1. Secular increase in the number of skilled workers throughout 20th century causing steady skill-biased technical change.
  2. Acceleration in the increase in the number of skilled workers over the past 25 years.
  3. Large increase in the number of unskilled workers available to be employed in the factories giving the 19th century.
- Strong endogenous bias can by itself explain major factor price movements.
- Is an upward sloping relative demand therefor labor reasonable?



The dynamics of the relative wage of skilled workers in response to an increase in the supply of skills with endogenous skill-biased technical change.

## Implications for Labor-Augmenting Technical Change

- Capital deepening combined with this result also has implications for the form of technical change in the aggregate economy.
- Existing evidence suggests that elasticity of substitution between capital and labor is less than 1.
- Therefore: capital deepening should lead to labor-augmenting technical change.
- However, typically not enough to keep interest rate and factor shares constant.

## Another Application: Cross-country Income Differences

- Many less developed countries (LDCs) use technologies developed in the U.S. and other OECD economies (the North).
- Directed technical change increases these concerns: it implies that technologies will be designed to make optimal use of the conditions and factor supplies in the North and will be *inappropriate* to the LDCs' needs (Acemoglu and Zilibotti, QJE 2001).

## Cross-country Income Differences

- Suppose that the model outlined above applies to a country I refer to as “the North” (either the U.S. or all the OECD countries as a whole).
- Suppose also that there are a set of LDCs in this world economy, each with with  $L'$  unskilled workers and  $H'$  skilled workers.

$$\frac{H'}{L'} < \frac{H}{L}.$$

- Assume that these LDCs will use the technologies developed in the North.

## Another Application: Cross-country Income Differences

- Now contrast two situations
  1. New technologies are developed for the Northern market (e.g., because of weak property rights in the LDCs).
  2. New technologies are developed for the world market.
- Irrespective the elasticity of substitution, new technologies will be more skill-biased when they are developed for the Northern market.
- **Cross-country Differences Result:** when new technologies are developed for the Northern market, there will be a larger income gap between the North and the South than would have been the case with random technologies or technologies directed at the world market.

## State Dependence in Innovation

- The innovation possibilities frontier used so far: “lab equipment” specification, no externalities, no scarce factors.
- Alternative: labor, skilled labor or scientists in R&D.
- Need some type of externalities, “building on the shoulders of giants,” to generate endogenous growth (though endogenous growth not central for the results here).
- This then also introduces the possibility of “state dependence in research” (of the relative state of knowledge).
  - State of knowledge in one sector may benefit (reduce innovation costs in) that sector more than the other.
  - With the “lab equipment” formulation, relative R&D costs are independent of past R&D.

## State Dependence in Innovation (continued)

- Consider the alternative innovation possibilities frontier

$$\dot{N}_L = \eta_L N_L^{(1+\delta)/2} N_Z^{(1-\delta)/2} S_L \text{ and } \dot{N}_Z = \eta_Z N_L^{(1-\delta)/2} N_Z^{(1+\delta)/2} S_Z,$$

for some  $\delta \leq 1$  where  $S$  scientists allocated to different technologies.

- In this specification,  $\delta$  measures the degree of state-dependence: when  $\delta = 0$ , there is no state-dependence
- When  $\delta = 1$ , there is “extreme” state-dependence: state of knowledge in one sector only benefits future innovations in that sector.

## State Dependence: Parallel Results

- Similar analysis shows that in this case long-run endogenous technologies are given by:

$$\frac{N_Z}{N_L} = \eta^{\frac{\sigma}{1-\delta\sigma}} \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\epsilon}{1-\delta\sigma}} \left( \frac{Z}{L} \right)^{\frac{\sigma-1}{1-\delta\sigma}}. \quad (20)$$

- Identical results to the previous setup when  $\delta = 0$ .

**Result 1' (Weak Endogenous Bias):**  $(N_Z/N_L)^{\frac{\sigma-1}{\sigma}}$  is increasing in  $Z/L$  as long as  $\sigma$  is not equal to 1.

- **Result 2** and **Result 3** also apply identical to before.
- Dynamics: stability is no longer guaranteed; necessary to have  $\sigma < 1/\delta$ .

## Strong Endogenous Bias

- Strong endogenous bias result strengthened:

$$\frac{w_Z}{w_L} = \eta^{\frac{\sigma}{1-\delta\sigma}} \left( \frac{1-\gamma}{\gamma} \right)^{\frac{(1-\delta)\epsilon}{1-\delta\sigma}} \left( \frac{Z}{L} \right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}. \quad (21)$$

**Result 4 (Strong Endogenous Bias):** If  $\sigma > 2 - \delta$ , then  $w_Z/w_L$  is increasing in  $Z/L$ .

## Results with Extreme State Dependence

- What happens when  $\delta = 1$ ?
- We obtain the results conjectured by Kennedy: directed technical change serves to equate factor shares.

$$\frac{s_Z}{s_L} \equiv \frac{w_Z Z}{w_L L} = \eta^{-1}. \quad (22)$$

- Moreover, for stability we need  $\sigma < 1$ —i.e., the factors need to be gross complements.

## Harrod Neutral Technical Change

- Next, suppose that  $Z$  is capital, and there is capital accumulation.
- For all  $\delta < 1$ , there is no balance growth path; because technical change not Harrod neutral.
- But if  $\delta = 1$  and capital and labor are gross complements, we obtain the result that the long run all technical change is Harrod neutral.
- Full analysis in Acemoglu (JEEA, 2003).

## Labor-Augmenting and Skill-Biased Technical Change?

- How do we reconcile extreme state-dependence for capital and labor,  $\delta = 1$ , with limited state-dependence, i.e.  $\delta < 1$ , between skilled and unskilled labor?
- It is possible to write a model with these features.
- Reasoning: state-dependence for general-purpose developments which are for labor or capital.
- Then, technologies for labor are adapted to skilled or unskilled labor, and here there is no state-dependence.
- Quite ad hoc.

## **More General Model: Acemoglu-Guerrieri, Non-Balanced Endogenous Growth**

- Framework for analysis of a growth path with constant aggregate growth, costs and capital share and interest rate, but non-balanced growth.
- Also technical change both labor-augmenting and skill-biased.
- Allow population growth

## The Environment

- Representative consumers with preferences

$$\int_0^{\infty} \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to the budget constraint

$$\dot{K} + C + X_H + X_L = Y$$

where  $X_H$  and  $X_L$  are R&D expenditures.

- Final good produced by combining a skill-intensive and a labor-intensive aggregate:

$$Y = \left[ \gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (23)$$

## The Environment (continued)

- Sectoral production functions (slightly more general than before)

$$Y_H = \left( \int_0^{M_H} y_H(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} \quad \text{and} \quad Y_L = \left( \int_0^{M_L} y_L(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$$

where the elasticity of substitution between products:  $\nu > 1$ .

- $M_H$  and  $M_L$  are the technology terms.  
 $M_H$ : number of labor-intensive goods.  
 $M_L$ : number of skill-intensive goods.
- The case of interest

$$\varepsilon < 1$$

to have a short-run elasticity of substitution between capital and labor  $< 1$ .

## The Environment (continued)

- Intermediate goods, supplied by monopolists, and produced as

$$y_H(i) = h(i)^\eta k_H(i)^{1-\eta} \quad (24)$$

and

$$y_L(i) = l(i)^\alpha k_L(i)^{1-\alpha} \quad (25)$$

- Focus on

$$\eta > \alpha$$

- Unskilled workers more “capital dependent” (similar to Beaudry and Green), but given  $\varepsilon < 1$ , there will always be *capital skill complementarity*.

## The Environment (continued)

- Market clearing for skilled and unskilled labor and for capital implies

$$\int_0^{M_H} h(i) di = H$$

$$\int_0^{M_L} l(i) di = L$$

and

$$\int_0^{M_H} k_H(i) di + \int_0^{M_L} k_L(i) di = K_H + K_L = K$$

- Assume exogenous population growth:

$$\frac{\dot{L}}{L} = \frac{\dot{H}}{H} = n$$

## The Environment (continued)

- Innovation possibilities frontier:

$$\dot{M}_H = b_H M_H^{-\varphi} X_H \quad (26)$$

$$\dot{M}_L = b_L M_L^{-\varphi} X_L \quad (27)$$

- More general than the extreme state dependence formulation above and also allows for population growth.

## Analysis

- The usual Euler equation

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho) \quad (28)$$

- Consumer maximization:

$$p \equiv \frac{p_H}{p_L} = \frac{\gamma}{1 - \gamma} \left( \frac{Y_H}{Y_L} \right)^{-\frac{1}{\varepsilon}} \quad (29)$$

- Set the price index as numeraire:

$$[\gamma^\varepsilon p_H^{1-\varepsilon} + (1 - \gamma)^\varepsilon p_L^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 1 \quad (30)$$

## Analysis (continued)

- Since symmetry within each sector, the same level of employment of factors and capital in each good within a sector, hence

$$Y_H = M_H^{\frac{1}{\nu-1}} H^\eta K_H^{1-\eta}$$

$$Y_L = M_L^{\frac{1}{\nu-1}} L^\alpha K_L^{1-\alpha}$$

- Aggregate production function:

$$Y = \left[ \gamma \left( M_H^{\frac{1}{\nu-1}} H^\eta K_H^{1-\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) \left( M_L^{\frac{1}{\nu-1}} L^\alpha K_L^{1-\alpha} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

## Analysis (continued)

- The value of a firm which becomes the monopolist of a new good respectively in the two sectors:

$$V_s(t) = \int_t^\infty \exp \left[ - \int_t^\tau r(\omega) d\omega \right] \pi_s(\tau) d\tau \quad (31)$$

for  $s = L$  and  $H$ .

- Finally, free entry into R&D implies:

$$V_L = \frac{M_L^\varphi}{b_L} \text{ and } V_H = \frac{M_H^\varphi}{b_H}$$

## Equilibrium and Comparative Statics Given State Variables

- Take the state variables  $K$ ,  $M_H$  and  $M_L$  as given.
  - Short-run capital-labor elasticity  $< 1$ :

$$\frac{d \ln s_K}{d \ln K} < 0$$

where  $s_K \equiv rK/Y$ . Key assumption  $\varepsilon < 1$ .

- Capital-labor complementarity

$$\frac{d \ln \omega}{d \ln K} < 0$$

where  $\omega \equiv w_H/w_L$ . Key assumption  $\eta > \alpha$  given  $\varepsilon < 1$ .

## Equilibrium and Comparative Statics (continued)

- Factor demand curves downward sloping in the short run:

$$\frac{d \ln \omega}{d \ln L} > 0$$

and

$$\frac{d \ln \omega}{d \ln H} < 0$$

- Technology

$$\frac{d \ln \omega}{d \ln M_H} < 0 \text{ and } \frac{d \ln \omega}{d \ln M_L} > 0$$

- Moreover,  $M_H/M_L$  capital biased and  $M_L/M_H$  capital augmenting.

## Asymptotic and Constant Growth Paths

- Asymptotic path: equilibrium path as  $t \rightarrow \infty$ .
- Defining a Constant Growth Path (CGP) as:

$$\lim_{t \rightarrow \infty} \frac{\dot{C}(t)}{C(t)} = \text{constant}$$

- Define:

$$\frac{\dot{K}_H}{K_H} \equiv z_H, \quad \frac{\dot{K}_L}{K_L} \equiv z_L, \quad \frac{\dot{Y}_H}{Y_H} \equiv g_H$$

$$\frac{\dot{Y}_L}{Y_L} \equiv g_L, \quad \frac{\dot{M}_H}{M_H} \equiv m_H, \quad \frac{\dot{M}_L}{M_L} \equiv m_L.$$

- Assume

$$\zeta \equiv (\nu - 1)(1 + \varphi) > \frac{1}{\eta}$$

- If in addition  $\varepsilon < 1$ , then there are no asymptotic explosive

## Constant Growth Path (Existence)

- In CGP, free-entry conditions for R&D imply:

$$g_H - \frac{1}{\varepsilon} (g_H - g) - (\varphi + 1) m_H = 0$$

and

$$g_L - \frac{1}{\varepsilon} (g_L - g) - (\varphi + 1) m_L = 0$$

- Combining with previous results yields:
  - There exists a *unique* asymptotic equilibrium with constant interest rate, constant share of capital, and the rate of growth of output, capital stock and consumption equal to each other.

## Constant Growth Path (Characterization)

- The case of interest for us is  $\varepsilon < 1$  and  $\eta > \alpha$ . Then, we have:

$$g = z_H = g_H$$

and

$$z_H = \frac{\eta\zeta}{\eta\zeta - 1}n$$

- Moreover:

$$m_H = \frac{z_H}{\varphi + 1} \text{ and } m_L = \frac{z_L}{\varphi + 1}$$

and

$$z_L = \frac{\zeta [\alpha (\eta\zeta - 1) (\varepsilon - 1) + \eta\zeta]}{(\eta\zeta - 1) [(\varepsilon - 1) (\alpha\zeta - 1) + \zeta]}n$$

where recall that

$$\zeta \equiv (\nu - 1) (1 + \varphi) > \frac{1}{\eta}$$

## Additional CGP Equilibrium Implications

- Interest rate, consumption growth rate and share of capital constant in the limiting equilibrium.
  - But the two sectors grow at different rates.

- Given  $\varepsilon < 1$  and  $\eta > \alpha$ :

$$\frac{\dot{\omega}}{\omega} > 0$$

- Therefore, technology steadily (and endogenously) becoming more and more skill biased.
- Moreover, asymptotic equilibrium involves  $m_L > 0, m_H > 0$ , so “capital-augmenting” technical change, except in the limit becomes Harrod neutral.